P-CuReD – a fast wavefront reconstruction algorithm for XAO with pyramid WFS

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Outline

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Problem

XAO (EPICS on E-ELT)

- $D = 42$ m telescope
- pyramid WFS with 200×200 subapertures, without / with circular modulation $\alpha = 4\lambda/D$
- frame rate 3 kHz, time for reconstruction: 0.3 ms

Task

- To determine the unknown wavefront ϕ from pyramid WFS data S_x, S_y

$$S_x = Q_x\phi, \qquad S_y = Q_y\phi \tag{1}$$

with Q_x, Q_y – nonlinear singular integral operators.

Linearized forward models

- To linearize Q_x, Q_y, we use assumptions: roof approximation, infinite telescope size, small wavefront perturbations (closed loop)

$$S_x^n(x,\cdot) = \phi(x,\cdot) * \frac{1}{\pi x}, \tag{2}$$

$$S_x^l(x,\cdot) = \phi(x,\cdot) * \frac{\mathrm{sinc}(\alpha_\lambda x)}{\pi x}, \tag{3}$$

$$S_x^c(x,\cdot) = \phi(x,\cdot) * \frac{J_0(\alpha_\lambda x)}{\pi x}, \tag{4}$$

$\{n, l, c\}$ – no / linear / circular modulation case,
$\alpha_\lambda = 2\pi\alpha/\lambda, \quad \alpha = b\lambda/D$ – modulation radius,
b – positive integer, J_0 – zero-order Bessel function of the first kind.

Background

Vérinaud'04 'On the nature of the measurements ...'

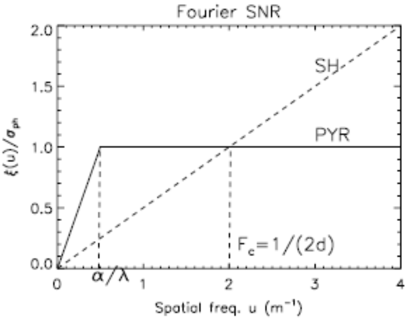


Fig. 3. Fourier SNR curves for the SHS with quad-cells (dashed line) and for the Pyramid sensor (solid line). Sub-aperture size $d = 0.25$ m.

P-CuReD

P-CuReD = Data Preprocessing + CuReD

Two-step method

1. **Data preprocessing:** Transform P-WFS data to SH-like data according to the analytical relation in the Fourier domain.
2. **CuReD:** To the modified data apply the CuReD – very efficient reconstructor for SH-WFS data.

   Details on CuReD:
   Matthias Rosensteiner, Poster 13208,
   Fast wavefront reconstruction with the CuReD algorithm

Step 1: Data preprocessing

- Representation of sensor data in the Fourier domain (Vérinaud'04)

$$(\mathcal{F}S_{pyr})(u) = (\mathcal{F}\phi)(u) \cdot g_{pyr}(u) \cdot \mathrm{sinc}(du) \quad (5)$$

$$(\mathcal{F}S_{sh})(u) = (\mathcal{F}\phi)(u) \cdot g_{sh}(u) \cdot \mathrm{sinc}(du) \quad (6)$$

u – spatial frequency, d – subaperture size.

- Fourier domain relation between the two sensors

$$(\mathcal{F}S_{sh})(u) = (\mathcal{F}S_{pyr})(u) \cdot g_{sh/pyr}(u), \qquad g_{sh/pyr}(u) := \frac{g_{sh}(u)}{g_{pyr}(u)}. \quad (7)$$

- Fourier convolution theorem $\rightarrow$ relation between the two sensors in the space domain

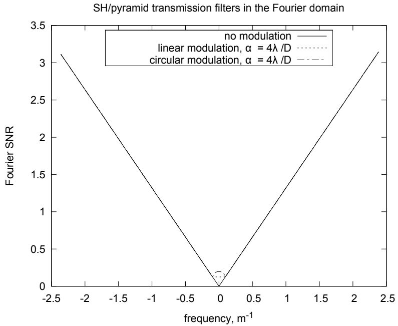
$$S_{sh}(x) = \frac{1}{\sqrt{2\pi}} S_{pyr}(x) * \underbrace{\left(\mathcal{F}^{-1} g_{sh/pyr}\right)(x)}_{p_{sh/pyr}(x)}. \quad (8)$$

Step 1: SH-Pyr transmission filters

$$g^{n}_{sh/pyr}(u) = 2\pi du\,\mathrm{sgn}(u), \forall u \in [-u_{cut}, u_{cut}] \tag{9}$$

$$g^{l}_{sh/pyr}(u) = \begin{cases} 2\pi du\,\mathrm{sign}(u), & |u| > u_{mod}, \\ 2\pi du_{mod}, & |u| <= u_{mod}, \end{cases} \tag{10}$$

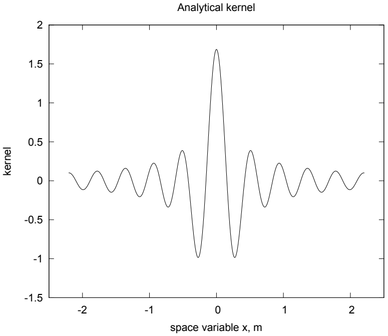
$$g^{c}_{sh/pyr}(u) = \begin{cases} 2\pi du\,\mathrm{sgn}(u), & |u| > u_{mod}, \\ \frac{\pi^2 du}{\arcsin(u/u_{mod})}, & |u| \leq u_{mod}, \end{cases} \tag{11}$$



Step 1: Space domain kernels

$$p^{n}_{sh/pyr}(x) = \frac{\pi}{d}\,\mathrm{sinc}\left(\frac{\pi x}{d}\right) - \frac{\pi}{2d}\mathrm{sinc}^2\left(\frac{\pi x}{2d}\right). \tag{12}$$

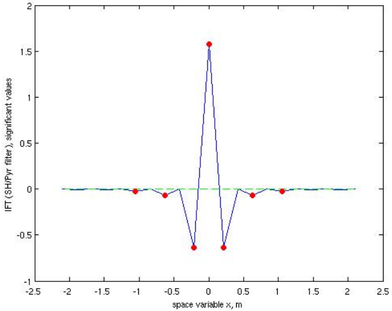
$$p^{l}_{sh/pyr}(x) = \frac{\pi}{d}\,\mathrm{sinc}\left(\frac{\pi x}{d}\right) + \frac{2\pi d b^2}{D^2}\mathrm{sinc}^2\left(\frac{\pi x b}{D}\right) - \frac{\pi}{2d}\mathrm{sinc}^2\left(\frac{\pi x}{2d}\right). \tag{13}$$



Step 1: Convolution

- Convolve S_x row-wise and S_y column-wise with 1d kernel $p_{sh/pyr}$

$$S_{sh}(x) = \frac{1}{\sqrt{2\pi}} S_{pyr}(x) * p_{sh/pyr}(x). \quad (14)$$

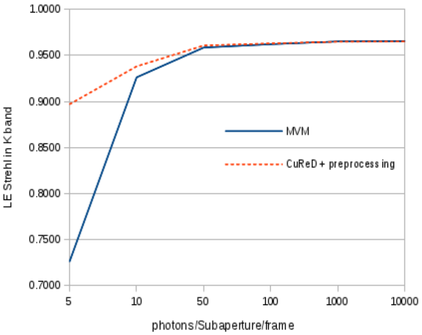


- Computationally very cheap, highly parallelizable and pipelinable.

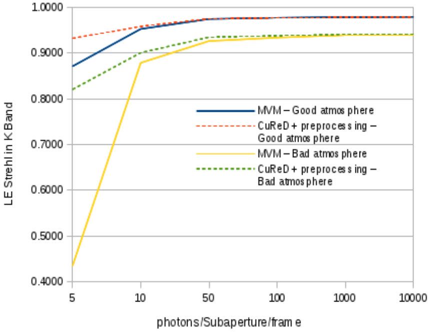
LE Strehl

ESO end-to-end simulator OCTOPUS

LE Strehl in K band: MVM and P-CuReD vs. the detected NGS photon flux.



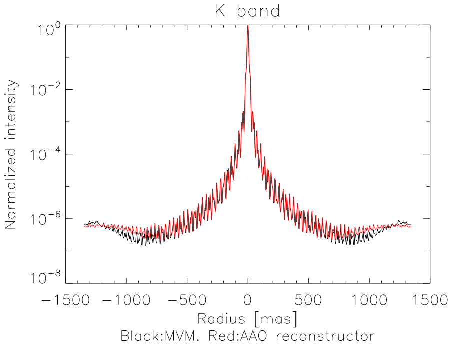
ESO median atmosphere



ESO bad/good atmospheres

LE PSF

LE PSF in the K-band for median atmosphere and high-flux case.



Complexity: P-CuReD vs MVM

- Data preprocessing requires $26N$ operations.
- CuReD requires $20N$ operations.
- Both steps are highly parallelizable and pipelinable.

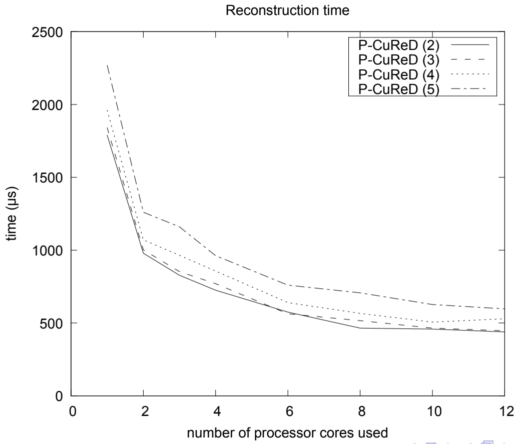
	P-CuReD	MVM
Complexity	$46n$	$4n^2$
Number of megaFLOP for $\sim$ 29600 actuators to be performed within $0.3ms$	1.36	3500
Percentage of MVM flop	0.04%	100%
Parallelizability and pipelinability	yes	yes

Speed of the C-prototype

System configuration:

Intel(R) Xeon(R) CPU X5650 @ 2.67GHz, 12 Cores (dual hexacore)

MVM: 402 ms



Paper

More quality results in:

Iu. Shatokhina, A. Obereder, M. Rosensteiner, R. Ramlau.
Preprocessed cumulative reconstructor with domain decomposition:
a fast wavefront reconstruction method for pyramid wavefront sensor.
Applied Optics, 52(12), 2640–2652 (2013).

- Derivation of analytical kernel for different modulation scenarios
- SE Strehl (convergence)
- LE Strehl for non-modulated case
- Noise propagation analysis

Summary

- P-CuReD for P-WFS with / without modulation.
- Linear complexity $\mathcal{O}(N)$.
- Strehl ratios are the same as / better than MVM.
- PSF is suitable for extrasolar planet search.
- Method suits the XAO requirements on ELT both wrt quality and speed.

Thanks for your attention!

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