A finite element–wavelet hybrid algorithm for atmospheric tomography

Mykhaylo Yudytskiy, Tapio Helin, Ronny Ramlau

May 31, 2013

Florence, Italy

This research was partially supported by the Austrian Science Fund (FWF): W1214-N15, project DK8.

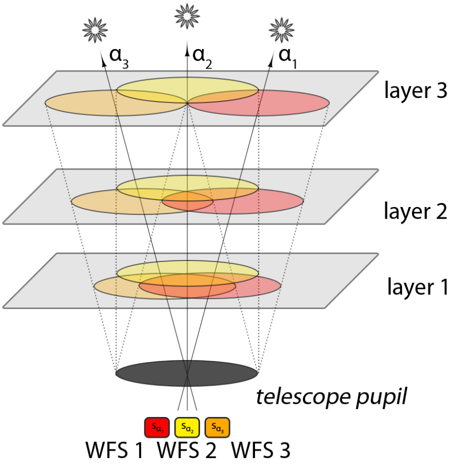
Outline

- Wavelets
- Finite element–wavelet algorithm
- Results: MCAO
- Results: LTAO

Atmospheric tomography

- Multi Conjugate AO (MCAO)
- Laser Tomography AO (LTAO)
- Multi Object AO (MOAO)

  - use several guidestars
  - goal: quality in the field of view

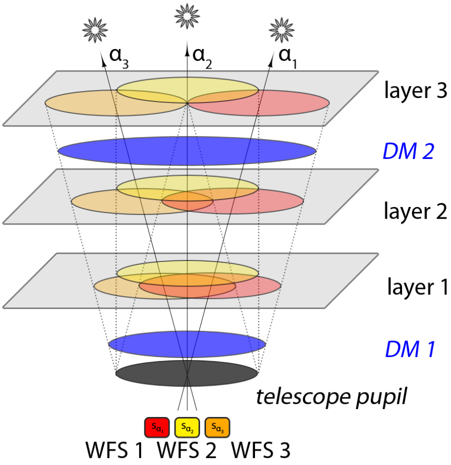


Atmospheric tomography: WFS measurements $\rightarrow$ layers

Atmospheric tomography

- **Multi Conjugate AO (MCAO)**
- Laser Tomography AO (LTAO)
- Multi Object AO (MOAO)

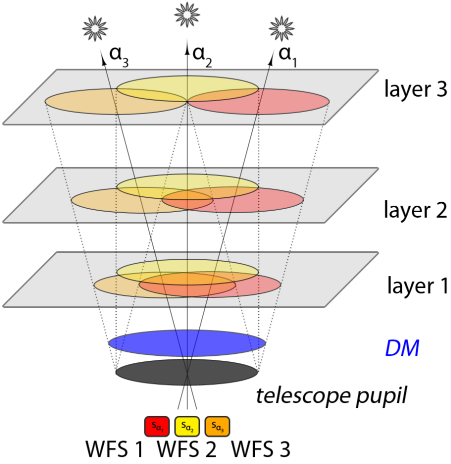
  - use several guidestars
  - goal: quality in the field of view



Atmospheric tomography: WFS measurements $\rightarrow$ layers

Atmospheric tomography

- Multi Conjugate AO (MCAO)
- **Laser Tomography AO (LTAO)**
- Multi Object AO (MOAO)

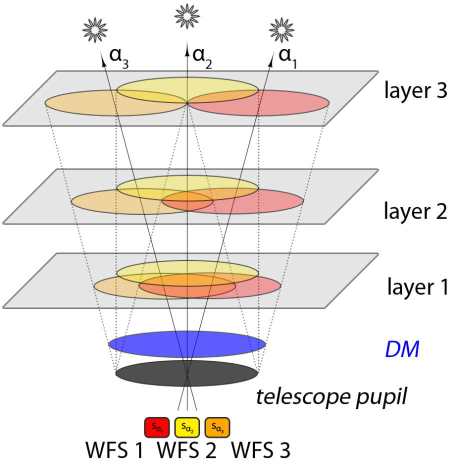


- use several guidestars
- goal: quality in the field of view

Atmospheric tomography: WFS measurements $\rightarrow$ layers

Atmospheric tomography

- Multi Conjugate AO (MCAO)
- **Laser Tomography AO (LTAO)**
- Multi Object AO (MOAO)



- use several guidestars
- goal: quality in the field of view

$$A\phi = s$$

atmospheric tomography operator ↗ ↑ turbulence layers ↖ noisy measurements

Atmospheric tomography: WFS measurements → layers

Parametrization of layers with wavelets

Concept: use **wavelets** to represent **turbulence layers**

Wavelets:

- a way to represent and analyze signals
- used in JPEG compression

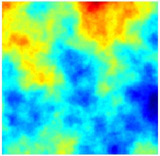
Parametrization of layers with wavelets

Concept: use **wavelets** to represent **turbulence layers**

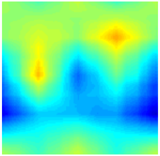
Wavelets:

- a way to represent and analyze signals
- used in JPEG compression

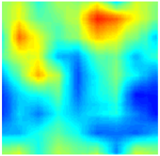
Wavelets decomposition of a layer:



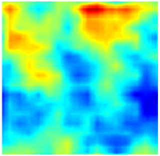
turb. layer
16,384 coeff



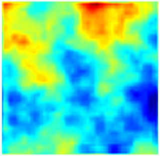
scales 0,1
16 coeff



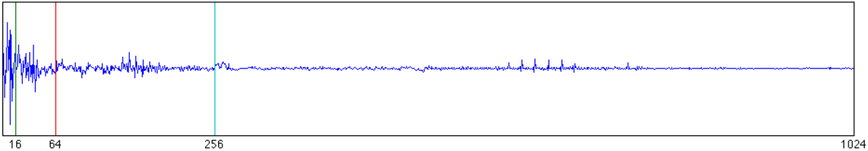
scales 0,1,2
64 coeff



scales 0,..,3
256 coeff



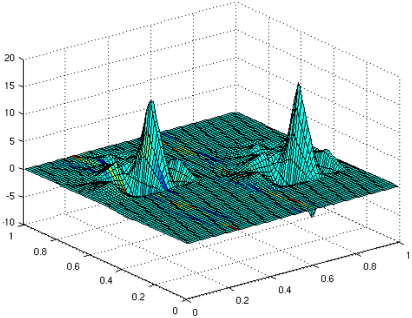
scales 0,..,4
1024 coeff



Wavelet properties

Advantages of wavelets:

- multiscalar structure
  - good approximative properties

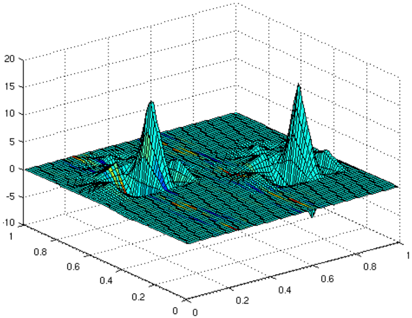


two Daubechies 3 wavelets

Wavelet properties

Advantages of wavelets:

- multiscalar structure
  - good approximative properties
- compact support
  - discrete wavelet transform (DWT)
    DWT is $\mathcal{O}(n)$, parallelizable!

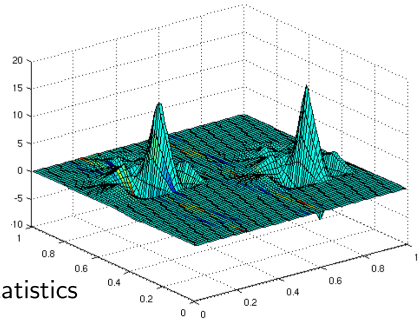


two Daubechies 3 wavelets

Wavelet properties

Advantages of wavelets:

- multiscalar structure
  - good approximative properties
- compact support
  - discrete wavelet transform (DWT)
    DWT is $\mathcal{O}(n)$, parallelizable!
- useful properties in frequency domain
  - efficient representation of turbulence statistics

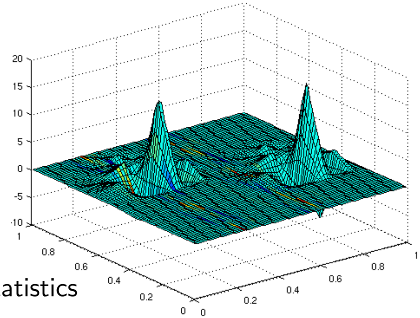


two Daubechies 3 wavelets

Wavelet properties

Advantages of wavelets:

- multiscalar structure
  - good approximative properties
- compact support
  - discrete wavelet transform (DWT)
    DWT is $\mathcal{O}(n)$, parallelizable!
- useful properties in frequency domain
  - efficient representation of turbulence statistics



two Daubechies 3 wavelets

Kolmogorov power law:

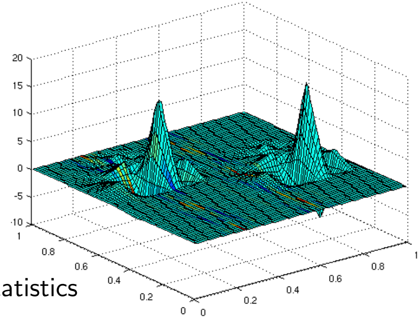
$$C_\phi = c\mathcal{F}^{-1} M \mathcal{F}$$

$$(Mf)(\xi) = |\xi|^{-11/3} f(\xi)$$

Wavelet properties

Advantages of wavelets:

- multiscalar structure
  - good approximative properties
- compact support
  - discrete wavelet transform (DWT)
    DWT is $\mathcal{O}(n)$, parallelizable!
- useful properties in frequency domain
  - efficient representation of turbulence statistics



two Daubechies 3 wavelets

Kolmogorov power law:

$$C_\phi = c\mathcal{F}^{-1} M \mathcal{F} \qquad C_\phi \simeq c W^{-1} D W$$

$$(Mf)(\xi) = |\xi|^{-11/3} f(\xi) \qquad D = \mathrm{diag}(..., 2^{-\frac{11}{3}j}, ...)$$

j... wav scale

Minimum variance turbulence profile estimation methods

Minimum variance turbulence profile estimate:

inverse noise covariance, inverse turbulence covariance

$$\underbrace{\overbrace{(A^* C_\eta^{-1}}^{\text{inverse noise covariance}} A}_{\text{atmospheric tomography}} + \overbrace{C_\phi^{-1}}^{\text{inverse turbulence covariance}})\underbrace{\phi}_{\text{turbulence layers}} = A^* C_\eta^{-1} \underbrace{s}_{\text{noisy measurements}}$$

atmospheric tomography, turbulence layers, noisy measurements

Alternatives:

the 3-step-approach[1] (e.g., Kaczmarz iteration, Gradient method, cg–method)

[1] Ramlau, Rosensteiner, Saxenhuber, Obereder

Minimum variance turbulence profile estimation methods

Solution methods to $(A^* C_\eta^{-1} A + C_\phi^{-1})\phi = A^* C_\eta^{-1} s$:

		atmospheric tomography	turbulence covariance	precond.	cost
	MVM	–	–	–	$\mathcal{O}(n^2)$
(P)CG Based	bilinear domain (MG-PCG[1])	sparse	biharmonic approx	multigrid	$\mathcal{O}(n^{3/2})$
	Fourier domain (FD-PCG[2])	very sparse	exact, diagonal	Fourier approx	$\mathcal{O}(n \log n)$
	FrIM–3D[3]	sparse	fractal approx	diagonal	$\mathcal{O}(n)$
	Finite Element–Wavelet				

◦ PCG = preconditioned conjugate gradient

[1]Gilles, Vogel, Ellerbroek
[2]Yang, Vogel, Ellerbroek
[3]Tallon, Béchet, Thiébaut et al.

Minimum variance turbulence profile estimation methods

Solution methods to $(A^* C_\eta^{-1} A + C_\phi^{-1})\phi = A^* C_\eta^{-1} s$:

		atmospheric tomography	turbulence covariance	precond.	cost
	MVM	–	–	–	$\mathcal{O}(n^2)$
(P)CG Based	bilinear domain (MG-PCG[1])	sparse	biharmonic approx	multigrid	$\mathcal{O}(n^{3/2})$
(P)CG Based	Fourier domain (FD-PCG[2])	very sparse	exact, diagonal	Fourier approx	$\mathcal{O}(n \log n)$
(P)CG Based	FrIM–3D[3]	sparse	fractal approx	diagonal	$\mathcal{O}(n)$
(P)CG Based	Finite Element–Wavelet	sparse			

- PCG = preconditioned conjugate gradient

[1] Gilles, Vogel, Ellerbroek
[2] Yang, Vogel, Ellerbroek
[3] Tallon, Béchet, Thiébaut et al.

Minimum variance turbulence profile estimation methods

Solution methods to $(A^* C_\eta^{-1} A + C_\phi^{-1})\phi = A^* C_\eta^{-1} s$:

		atmospheric tomography	turbulence covariance	precond.	cost
	MVM	–	–	–	$\mathcal{O}(n^2)$
(P)CG Based	bilinear domain (MG-PCG[1])	sparse	biharmonic approx	multigrid	$\mathcal{O}(n^{3/2})$
	Fourier domain (FD-PCG[2])	very sparse	exact, diagonal	Fourier approx	$\mathcal{O}(n \log n)$
	FrIM–3D[3]	sparse	fractal approx	diagonal	$\mathcal{O}(n)$
	Finite Element–Wavelet	sparse	diagonal approx		

- PCG = preconditioned conjugate gradient

[1] Gilles, Vogel, Ellerbroek
[2] Yang, Vogel, Ellerbroek
[3] Tallon, Béchet, Thiébaut et al.

Minimum variance turbulence profile estimation methods

Solution methods to $(A^* C_\eta^{-1} A + C_\phi^{-1})\phi = A^* C_\eta^{-1} s$:

		atmospheric tomography	turbulence covariance	precond.	cost
	MVM	–	–	–	$\mathcal{O}(n^2)$
(P)CG Based	bilinear domain (MG-PCG[1])	sparse	biharmonic approx	multigrid	$\mathcal{O}(n^{3/2})$
	Fourier domain (FD-PCG[2])	very sparse	exact, diagonal	Fourier approx	$\mathcal{O}(n \log n)$
	FrIM–3D[3]	sparse	fractal approx	diagonal	$\mathcal{O}(n)$
	Finite Element–Wavelet	sparse	diagonal approx	diagonal	

- PCG = preconditioned conjugate gradient

[1] Gilles, Vogel, Ellerbroek
[2] Yang, Vogel, Ellerbroek
[3] Tallon, Béchet, Thiébaut et al.

Minimum variance turbulence profile estimation methods

Solution methods to $(A^* C_\eta^{-1} A + C_\phi^{-1})\phi = A^* C_\eta^{-1} s$:

		atmospheric tomography	turbulence covariance	precond.	cost
	MVM	–	–	–	$\mathcal{O}(n^2)$
(P)CG Based	bilinear domain (MG-PCG[1])	sparse	biharmonic approx	multigrid	$\mathcal{O}(n^{3/2})$
	Fourier domain (FD-PCG[2])	very sparse	exact, diagonal	Fourier approx	$\mathcal{O}(n \log n)$
	FrIM–3D[3]	sparse	fractal approx	diagonal	$\mathcal{O}(n)$
	Finite Element–Wavelet	sparse	diagonal approx	diagonal	$\mathcal{O}(n)$

◦ PCG = preconditioned conjugate gradient

[1] Gilles, Vogel, Ellerbroek
[2] Yang, Vogel, Ellerbroek
[3] Tallon, Béchet, Thiébaut et al.

Finite Element–Wavelet algorithm

Solve:

$$(W A^* C_\eta^{-1} A W^{-1} + \alpha D^{-1}) c = W A^* C_\eta^{-1} s$$

discrete wavelet transform $\mathcal{O}(n)$

atmospheric tomography bilinear basis (sparse)

diagonal operator wavelet basis

wavelet coefficients

Method handles:

- cone effect
- tip/tilt indetermination
- spot elongation

Finite Element–Wavelet algorithm

Solve:

$$\underbrace{\left(W A^* C_\eta^{-1} A W^{-1} + \alpha D^{-1} \right)}_{M} c = W A^* C_\eta^{-1} s$$

using a conjugate gradient (CG) based method.

Finite Element–Wavelet algorithm

Solve:

$$\underbrace{\left(W A^* C_\eta^{-1} A W^{-1} + \alpha D^{-1}\right)}_{M} c = W A^* C_\eta^{-1} s$$

using a conjugate gradient (CG) based method.

$$\boxed{\text{cost of the iterative method} = \text{cost of applying } M \cdot \# \text{ of iterations}}$$

Finite Element–Wavelet algorithm

Solve:

$$\underbrace{\left(W A^* C_\eta^{-1} A W^{-1} + \alpha D^{-1}\right)}_{M} c = W A^* C_\eta^{-1} s$$

using a conjugate gradient (CG) based method.

$$\boxed{\text{cost of the iterative method} = \text{cost of applying } M \cdot \#\text{ of iterations}}$$

cost of applying M

- M is **matrix–free** if applied sequentially!
- parallelizable w.r.t. layers, WFS

Finite Element–Wavelet algorithm

Solve:

$$\underbrace{\left(W A^* C_\eta^{-1} A W^{-1} + \alpha D^{-1} \right)}_{M} c = W A^* C_\eta^{-1} s$$

using a conjugate gradient (CG) based method.

$$\boxed{\text{cost of the iterative method} = \text{cost of applying } M \cdot \#\text{ of iterations}}$$

cost of applying M

- M is **matrix–free** if applied sequentially!
- parallelizable w.r.t. layers, WFS

\# of iterations

$\searrow$ warm restart

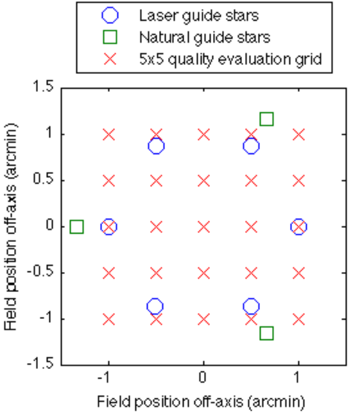
$\searrow$ preconditioner: Jacobi $= \mathrm{diag}(M)$

$\searrow$ cascadic multiscale

Simulations in OCTOPUS: MCAO

Configuration:

- Telescope aperture diameter: 42 m
- 6 laser guide stars (LGS)
    - 84×84 subapertures
- 3 natural guide stars (NGS)
    - 1 sensor with 2×2 subapertures
    - 2 sensors with 1×1 subapertures
- 3 DMs
    - at 0, 4000, 12,700 m
    - 9,296 active actuators

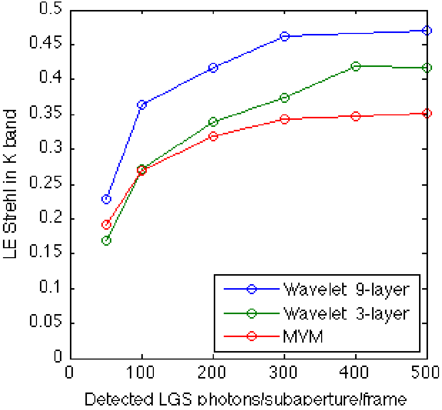


Simulated data:

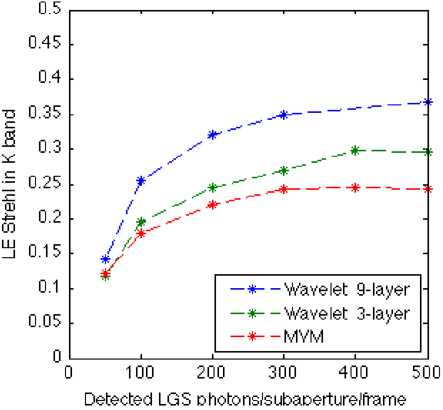
- OCTOPUS – official simulation tool of ESO
- 9 atmospheric layers
- quality evaluated in 25 directions

Quality results: MCAO

LGS flux:	50-500 photons/subap/frame, elongated spots
NGS flux:	500 photons/subap/frame
Method:	20 CG iterations



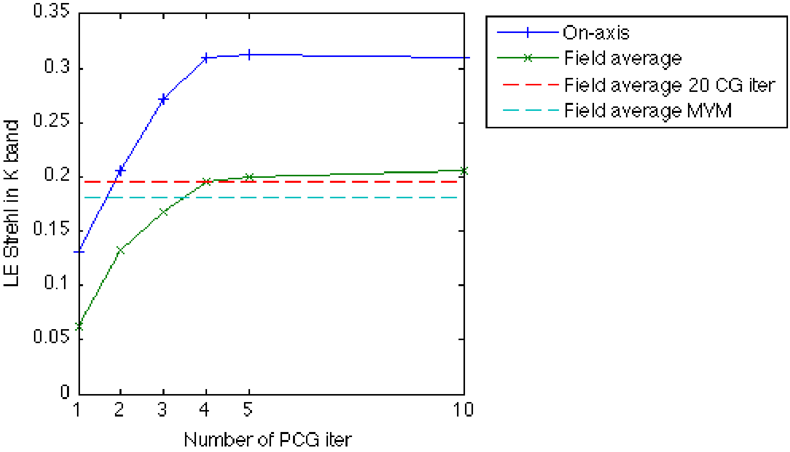
On-axis



Field average

Quality results: MCAO, with preconditioning

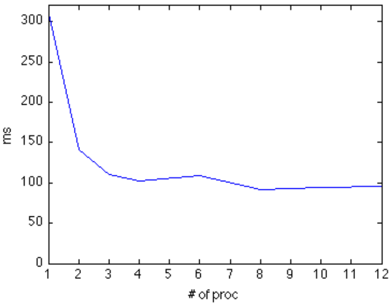
LGS flux:	100 photons/subap/frame, elongated spots
NGS flux:	500 photons/subap/frame
Method:	1-10 PCG iterations, 3-layer



Speed results: MCAO

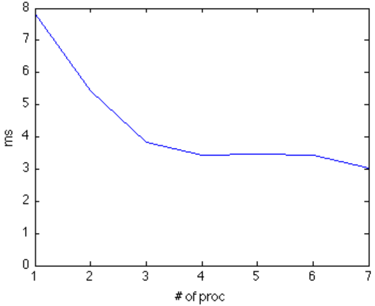
System configuration:

- Intel(R) Xeon(R) CPU X5650 @ 2.67GHz
- 12 Cores (dual hexacore)



MVM

92 ms



Finite Element–Wavelet
3-layer, PCG 4 iter
3.0 ms

- 7 cores used: 6 WFS + 1 core for TTS computation

Simulations in OCTOPUS: LTAO

Configuration:

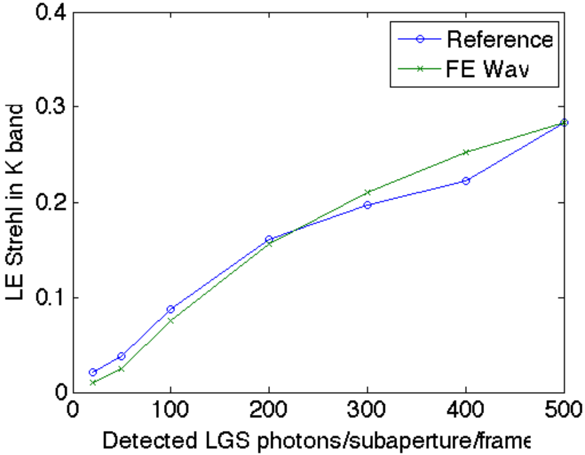
- Telescope aperture diameter: 42 m
- 6 laser guide stars (LGS)
  - 84×84 subapertures
  - in circle with 7.5 arcmin diam.
- 3 natural guide stars (NGS)
  - 84×84 subapertures
  - in circle with 10 arcmin diam.
- 1 DM
  - 5,402 active actuators

Simulated data:

- OCTOPUS – official simulation tool of ESO
- 9 atmospheric layers
- quality evaluated at the zenith

Quality results: LTAO

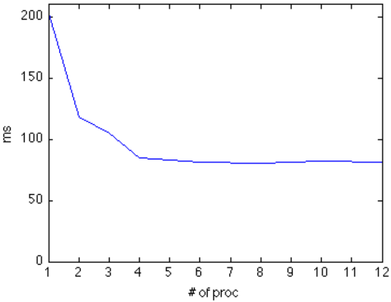
LGS flux:	20-500 photons/subap/frame, elongated spots
NGS flux:	300 photons/subap/frame
Method:	30-40 CG iterations



Speed results: LTAO/MOAO

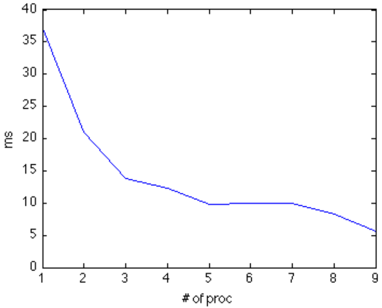
System configuration:

- Intel(R) Xeon(R) CPU X5650 @ 2.67GHz
- 12 Cores (dual hexacore)



MVM

80 ms



Finite Element–Wavelet
9-layer, PCG 4 iter
5.6 ms

- 9 cores used: 9 WFS / 9 layers

Summary and Outlook

Finite Element–Wavelet method

- CG-based
- globally $\mathcal{O}(n)$, parallelizable
- ◦ DWT $\mathcal{O}(n)$, parallelizable
- ◦ efficient representation of turbulence statistics
- ◦ cascadic multiscale

Outlook

- application to MOAO (open loop)
- combining preconditioning, multiscale and other ideas $\rightarrow$ reduce $\#$ of iter
- improving parallelization

Summary and Outlook

Finite Element–Wavelet method

- CG-based
- globally $\mathcal{O}(n)$, parallelizable
- ◦ DWT $\mathcal{O}(n)$, parallelizable
- ◦ efficient representation of turbulence statistics
- ◦ cascadic multiscale

Outlook

- application to MOAO (open loop)
- combining preconditioning, multiscale and other ideas $\rightarrow$ reduce $\#$ of iter
- improving parallelization

Thanks for your attention!