Wavefront Reconstruction for a Natural Guide Star Ground Layer Adaptive Optics System on the Giant Magellan Telescope

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Abstract. In this paper, we present a wavefront reconstruction paradigm for NGS GLAO systems. The conventional approach to reconstructing the wavefront for LGS GLAO systems is to have a number of LGSs in a ring outside the science field and simply average the individual wavefronts. This approach is not well-suited to NGS GLAO because the stars have an irregular distribution and varying magnitudes. In this paper, we derive covariance matrices for the wavefronts in different directions and the measurement noise. Using these covariance matrices, we are able to find the minimum-variance reconstructor and estimate the residual wavefront as a function of location in the field. This can be used to determine which guide stars produce the best correction. End-to-end simulations in YAO are run to estimate the expected performance of the NGS GLAO system for the Giant Magellan Telescope. We find that there is essentially full sky coverage.

1 Introduction

The Acquisition, Guider Wavefront Sensor (AGWS) system under design for the Giant Magellan Telescope has four sensors which are required to measure and compensate for distortions in the optical figure of the primary and secondary mirrors, and to make a coarse measurement of the telescope segment piston. The wavefront sensors are Shack-Hartmann guiding on Natural Guide Stars (NGSs) and are distributed on a radius of between 5' and 10'. If the active optics sensors operate with sufficient temporal and spatial bandwidth, they can also be used to compensate for the ground layer turbulence. This report investigates the performance of these active optics sensors at compensating for ground-layer turbulence. We use end-to-end simulations in YAO to determine the performance of such a system.

The remainder of the paper is distributed as follows. Section 2 describes a minimum-variance reconstructor for GLAO. The simulation parameters for the GMT GLAO system are presented in Sect. 3, with the corresponding simulation results in Sect. 4. Finally, conclusions are drawn in Sect. 5.

2 Wavefront reconstruction

The traditional way to reconstruct the wavefront in GLAO is to take the reconstructed wavefront from each WFS and average them together to produce a wavefront estimate. [2,3] In this paper, we present a minimum-variance reconstruction technique ideally suited to randomly located stars with varying magnitudes. This reconstructor also outperforms the standard reconstructor even in the case where the guide stars are evenly distributed and produce no measurement noise, as shown in Section 4.1.

The wavefront reconstruction problem can be described as follows. We have a number of noisy WFS measurements, s, and we wish to find the actuator commands, a, that minimizes the wavefront

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in one or more directions. The minimum-variance estimate of \( a \) is given by:\[5\]
\[
\hat{a} = \langle ax^T \rangle (ss^T)^{-1}s,
\]
and the corresponding wavefront error due to measurement noise and tomographic error is\[5\]
\[
\sigma^2_\phi = \langle aa^T \rangle - \langle ax^T \rangle ss^T (ss^T)^{-1} \langle ax^T \rangle.
\]

The square brackets are used to denote the expected value. Hence, an expression like \( \langle ss^T \rangle \) represents the covariance matrix of the noisy centroid measurements, \( s \). Vidal shows how to calculate the covariance matrix of the centroid measurements analytically.[4] Unfortunately, \( ss^T \) is not invertible, since there are a number of modes in the wavefront sensor that do not affect the reconstruction (the so-called “null modes” for classical AO). However, it can be inverted with some regularization. In addition, the covariance matrix for slope measurements has twice as many rows and columns as the covariance matrix of wavefront estimates.

Instead, we convert the slope measurements into wavefront estimates in each direction, \( x \). If \( H \) is the interaction matrix, such that
\[
s = Hx + n,
\]
where \( n \) is the measurement noise, then any conventional reconstructor, \( R \), can be used to compute the wavefront in each direction from \( s \):
\[
\hat{s} = Rs.
\]

Here, we use the reconstructors that penalize second derivatives in the wavefront, as suggested by Ellerbroek[6] and implemented in YAO:[7]
\[
R = (H^T H + \alpha L)^{-1} R
\]

where \( \alpha \) is a regularization parameter and \( L \) is a Laplacian penalization matrix.[6] Once we have the estimate of the wavefront in the direction of each wavefront sensor, we obtain the optimal DM commands:
\[
\hat{a} = \langle ax^T \rangle \left( \langle xx^T \rangle + \alpha (nn^T) R \right)^{-1} Rs,
\]
and we define the control matrix, \( M \), to be:
\[
M = \langle ax^T \rangle \left( \langle xx^T \rangle + \alpha (nn^T) R \right)^{-1} R.
\]

The wavefront error due to a combination of measurement noise and tomographic reconstruction in open-loop is
\[
\sigma^2_\phi = \langle aa^T \rangle - \langle ax^T \rangle xx^T (xx^T + \alpha nn^T R)^{-1} \langle ax^T \rangle.
\]

The remaining problem is how to calculate the covariance matrices.

The noise covariance matrix, \( \langle nn^T \rangle \), can be computed from first principles or using a modest number of open-loop simulations. We prefer to use the latter since this accounts for the precise behavior of the WFS, including non-linearities in the denominator, spot truncation and pixelization, etc. Unfortunately, the centroid estimates at low light levels are biased towards the center of the centroiding window, so the measurement error is not an unbiased zero-mean Gaussian random variable. It is found that Eq. (8) underestimates the open-loop wavefront error when the level of measurement noise is high, due to the bias in the centroid error that is not accounted for in the noise covariance matrix. In most existing AO systems this bias problem does not exist, as detectors typically have few pixels and the spots are driven towards (or near) the center of the detector. In future work, the centroiding algorithm will be replaced with the correlation algorithm,[8] which is not biased towards zero and has better noise performance.

The covariance matrix for the wavefront at two different locations, \( r_1 \) and \( r_2 \), and in two arbitrary directions, \( \theta_1 \) and \( \theta_2 \), depends on Fried’s parameter, \( r_0 \), the outer scale of turbulence, \( L_0 \) and the distribution of turbulence. A finite outer scale is required to avoid infinite values in the matrix. We assume
The altitude of the guide star, \( z \), the distance between two wavefronts at altitude layer \( k \) at height \( h(k) \). The covariance is given by:[1]

\[
\langle \phi(r_1, \theta_1), \phi^T(r_2, \theta_2) \rangle = c(r_0 f_0) \sum_{k=1}^{N_i} \epsilon(k) \left( 2\pi f_0 \delta_p(k) \right)^{5/6} K_{5/6}(2\pi f_0 \delta_p(k))
\]

(9)

where \( f_0 = 1/L_0 \) and \( K_{5/6} \) is the fractional Bessel function of the second kind of order 5/6. We define the distance between two wavefronts at altitude layer \( k \), as

\[
\delta_p(k) = |\rho_1(k) - \rho_2(k)|,
\]

(10)

where

\[
\rho_3(k) = (1 - h(k)/z_i) r_1 + h(k) \theta_1.
\]

(11)

The altitude of the guide star, \( z_i \), is infinite for a natural guide star. Finally, \( c \) is a constant

\[
c = \left( \frac{24}{5} \right)^{5/6} \left( \frac{6}{5} \right) \left( \frac{\Gamma(11/6)}{\Gamma(25/6)} \right)^{8/3}.
\]

(12)

where the symbol \( \Gamma \) represents the gamma function. In these calculations, the piston of the wavefront has not been removed; the piston term must be projected out of the reconstructor.

Finally, we calculate \( \langle \alpha x^2 \rangle \), the optimal DM commands given the estimated wavefront in the direction of each wavefront sensor. The covariance matrix between the wavefront at the WFS locations and 21 representative points in the science field is calculated, and then averaged over the points in the science field.

The reconstructor in Eq. (7) is the optimal open-loop reconstructor. However, the GLAO system runs in closed-loop, since the WFSs see the correction applied by the adaptive secondary mirror. We can operate in closed-loop using an open-loop reconstructor by applying so-called pseudo open-loop control.[6] In pseudo open-loop control, the error at time \( n \), \( u[n] \), is found by applying the reconstructor to the centroids that would be measured if no DM was present and subtracting the current commands:

\[
u[n] = M(s[n] + Ha[n]) - a[n].
\]

(13)

Eq (13) can be rewritten using the identity matrix, \( I \), as:

\[
u[n] = Ms[n] + (HM - I)a[n].
\]

(14)

The actuator commands are updated using a closed-loop control law, such as an integrator with a variable loop gain, \( g \):

\[
a[n + 1] = a[n] + ga[n].
\]

(15)

### 3 Description of adaptive optics system and operating conditions

The wavefront sensor (WFS) is a 24×24 Shack-Hartmann WFS with approximately 1-m subapertures. The camera selected is the Andor iXon Ultra 897. It has 512 × 512 pixels and the full frame can be read as fast as 56 Hz, and 109 Hz when binning the pixels by 2 × 2. The simulations presented in this paper were all run at 100 Hz with 2 × 2 binning. The simulations used 8 × 8 pixels per subaperture, each with a 0.3” angular extent. The camera can be cooled down to -100°C, to keep the dark current down to negligible levels. Subelectron read noise is obtained with electron multiplication. We assume a value of 0.5 \( e^- \) for the read noise. The noise excess factor inherent in electron multiplication, which is equivalent to a 50% reduction in quantum efficiency, was not taken into account in the simulations presented here. In future work, the excess noise factor will be considered.

The radial patrol range of each sensor is currently 5’ to 10’. One quadrant is nominally reserved for the phasing camera, but if the phasing camera is no longer needed for routine observing, it could...
be replaced with another GLAO WFS. There are two cases to be investigated: the narrow field and the wide field case. In the narrow field case, where we try to correct near the optical axis only, the shadow of M3 extends to a radius of 6', so only stars between 6' and 10' may be used. Both V- and R-bands are passed to the WFS. In the wide field mode, the probes may be brought in as close as 5' from the optical axis, since the wide field instruments are at the Gregorian focus and not affected by M3. There are substantial static wavefront and chromatic aberrations in the WFS spots because the WFS is located before the final element of the wide field corrector. The wavefront aberrations can be compensated using off-null reference centroids, but the spot size itself will not be affected. The chromatic aberration, however, will elongate all the spots, reducing the sensitivity. To reduce the effect of the spot elongation, the range of wavelengths to be used will be restricted to the R-band.

The photometric parameters are tabulated in Table 1.

Table 1. Photometric parameters used in the simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central wavelength</td>
<td>0.64 μm</td>
</tr>
<tr>
<td>Photometric zero point</td>
<td>4.0×10^{-12}</td>
</tr>
<tr>
<td>Sky background</td>
<td>18.9 (mag/square arcsec)</td>
</tr>
<tr>
<td>Quantum efficiency</td>
<td>90%</td>
</tr>
<tr>
<td>Optical throughput</td>
<td>60%</td>
</tr>
</tbody>
</table>

The atmospheric parameters are derived from the typical-typical profile for January 2008 from Goodwin[9] and are reproduced in Table 2 for convenience. This turbulence profile is pessimistic with respect to the fraction of turbulence located near the ground. The value of $r_0$ is 0.151 m at 500 nm and an outer scale of 60-m is assumed.

Table 2. Turbulence profile used in the simulations

<table>
<thead>
<tr>
<th>Elevation (m)</th>
<th>Turbulence fraction</th>
<th>Wind speed (m/s)</th>
<th>Wind direction (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>0.126</td>
<td>5.65</td>
<td>0.78</td>
</tr>
<tr>
<td>275</td>
<td>0.087</td>
<td>5.80</td>
<td>8.25</td>
</tr>
<tr>
<td>325</td>
<td>0.067</td>
<td>5.89</td>
<td>12.48</td>
</tr>
<tr>
<td>1250</td>
<td>0.350</td>
<td>6.64</td>
<td>32.50</td>
</tr>
<tr>
<td>4000</td>
<td>0.227</td>
<td>13.29</td>
<td>72.10</td>
</tr>
<tr>
<td>8000</td>
<td>0.068</td>
<td>34.83</td>
<td>93.20</td>
</tr>
<tr>
<td>13000</td>
<td>0.075</td>
<td>29.42</td>
<td>100.05</td>
</tr>
</tbody>
</table>

4 Simulation results

4.1 Noiseless ideal case

Initially, we run the ideal noiseless case, where there are three stars in an equilateral triangle at a radius of 6’ from the optical axis and there is a single on-axis science target. As a comparison, the simulation is repeated with simple wavefront sensor averaging. The results, tabulated in Table 3, show that the minimum-variance reconstructor is vastly superior to wavefront sensor averaging even in this ideal case.

4.2 Effect of measurement noise

A simple centroiding algorithm with pixel thresholding is used. In practice, an algorithm that is more noise-efficient and has less bias than the centroid, such as the correlation algorithm[8], should be used
Table 3. On-axis FWHM (mas) as a function of wavelength and wavefront reconstruction strategy

<table>
<thead>
<tr>
<th>Wavelength (µm)</th>
<th>0.440</th>
<th>0.550</th>
<th>0.640</th>
<th>0.790</th>
<th>1.215</th>
<th>1.654</th>
<th>2.179</th>
</tr>
</thead>
<tbody>
<tr>
<td>No correction</td>
<td>562.0</td>
<td>539.3</td>
<td>513.8</td>
<td>500.9</td>
<td>455.6</td>
<td>418.3</td>
<td>363.0</td>
</tr>
<tr>
<td>Averaged WFSs</td>
<td>541.8</td>
<td>516.9</td>
<td>504.8</td>
<td>485.9</td>
<td>422.2</td>
<td>363.2</td>
<td>297.7</td>
</tr>
<tr>
<td>minimum-variance reconstructor</td>
<td>489.7</td>
<td>456.1</td>
<td>430.1</td>
<td>390.8</td>
<td>321.2</td>
<td>262.1</td>
<td>205.2</td>
</tr>
</tbody>
</table>

instead. The FWHM as a function of guide star R-magnitude is tabulated in Table 4. Using brighter stars than 14th magnitude did not appreciably improve the performance. The results show that the FWHM begins to degrade when using stars fainter than 15th magnitude.

Table 4. On-axis FWHM (mas) as a function of wavelength and R-band guide star magnitude

<table>
<thead>
<tr>
<th>Wavelength (µm)</th>
<th>0.440</th>
<th>0.550</th>
<th>0.640</th>
<th>0.790</th>
<th>1.215</th>
<th>1.654</th>
<th>2.179</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>489.4</td>
<td>437.2</td>
<td>435.1</td>
<td>394.8</td>
<td>324.1</td>
<td>256.8</td>
<td>199.3</td>
</tr>
<tr>
<td>15</td>
<td>493.5</td>
<td>458.3</td>
<td>438.0</td>
<td>402.8</td>
<td>330.8</td>
<td>265.5</td>
<td>198.3</td>
</tr>
<tr>
<td>16</td>
<td>496.2</td>
<td>470.3</td>
<td>444.9</td>
<td>417.1</td>
<td>337.8</td>
<td>284.2</td>
<td>224.6</td>
</tr>
<tr>
<td>17</td>
<td>506.9</td>
<td>484.6</td>
<td>463.6</td>
<td>433.5</td>
<td>362.0</td>
<td>318.1</td>
<td>258.5</td>
</tr>
<tr>
<td>18</td>
<td>536.7</td>
<td>491.2</td>
<td>490.9</td>
<td>460.1</td>
<td>412.3</td>
<td>353.8</td>
<td>306.9</td>
</tr>
</tbody>
</table>

4.3 Sky coverage simulations

Simulated star fields were generated for the South Galactic pole using the Besancon star model in the following manner. First, the Besancon web interface (http://model.obs-besancon.fr/) was used to generate a simulated catalog of a 1 square degree region at the South Galactic Pole. This produced a list of stars with the correct distribution of magnitudes and colors. Then random positions within a one degree square were reassigned to each star in the input list. Stars within 10’ of the field center were selected for further processing by the GLAO simulation.

The sum of the tomographic and measurement noise error was calculated for each of 100 fields using Eq. (8). The results, displayed in Figure 1, show that the variation in error with field is small compared to the difference between using GLAO and not using it. Next, fifteen such random fields were selected and simulated. The simulations comprised of a total of 100 seconds of simulation time.

Fig. 1. Analytic wavefront error as a function of star field percentile for the on-axis correction case. The RMS wavefront error in the uncorrected case is 2.43 microns.
Fig. 2. A random asterism at the South Galactic Pole, excluding the stars within the central 5° radius. The size of the stars is indicative of their brightness as shown in the legend. The blue circles indicate the inner and outer search radii. The gray square represents the excluded region due to the phase sensing camera. There are red circles around the three stars selected as guide stars.

Fig. 3. V-band (left) and K-band (right) variability in FWHM across the science field for a 10' diameter field.

in 10 s blocks with a 30 s skip in between. An example of a star field is shown in Figure 2. We optimize the reconstruction of this star field over a 10' diameter circle. The FWHM across the field at V-band and K-band is displayed in Figure 3.
Figure 4 shows the improvement in FWHM as a function of wavelength and star field for a 10’ diameter field correction. It can be seen that the scatter in FWHM is small, showing that essentially the same performance is obtained for every field.

A comparison of the average performance of GLAO for a science field with a 0’, 5’ and 10’ diameter is shown in Figure 5.

5 Conclusion

The conventional way to compute the GLAO reconstructor is to take the average of the WFS measurements. In this paper, a minimum-variance reconstructor for N-GLAO is presented. Using end-to-end Monte Carlo simulations, we show that the minimum-variance reconstructor exhibits much improved performance to the conventional approach. Using a minimum-variance reconstructor and pseudo-open loop reconstruction, we show that it is possible to obtain improved image quality over a 10’ diameter field with 100% sky coverage.

Further work is needed to improve upon this initial work. The excess noise factor inherent in the electron multiplication of the CCD needs to be included. The centroiding algorithm will be replaced
by the correlation algorithm, in order to reduce the bias and error in the measurements. The parameters of the GLAO system need to be optimized; for example, the number and extent of the pixels in each subaperture, and the total number of subapertures. Finally, it may be possible to create a reconstructor that does not reduce the total wavefront error but instead minimizes the FWHM or another quantity of interest, such as encircled energy.

Acknowledgments

This work has been supported by the GMTO Corporation, a non-profit organization operated on behalf of an international consortium of universities and institutions: Astronomy Australia Ltd, the Australian National University, the Carnegie Institution for Science, Harvard University, the Korea Astronomy and Space Science Institute, the Smithsonian Institution, The University of Texas at Austin, Texas A&M University, University of Arizona and University of Chicago. This material is based in part upon work supported by AURA through the National Science Foundation under Scientific Program Order No. 10 as issued for support of the Giant Segmented Mirror Telescope for the United States Astronomical Community, in accordance with Proposal No. AST-0443999 submitted by AURA.

The Gemini Observatory is operated by the Association of Universities for Research in Astronomy, Inc., under a cooperative agreement with the NSF on behalf of the Gemini partnership: the National Science Foundation (United States), the Science and Technology Facilities Council (United Kingdom), the National Research Council (Canada), CONICYT (Chile), the Australian Research Council (Australia), Ministério da Ciência e Tecnologia (Brazil) and Ministerio de Ciencia, Tecnología e Innovación Productiva (Argentina).

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