Third AO4ELT Conference - Adaptive Optics for Extremely Large Telescopes
Florence, Italy. May 2013
ISBN: 978-88-908876-0-4
DOI: 10.12839/AO4ELT3.13218

FREQUENCY-BASED DESIGN OF ADAPTIVE OPTICS SYSTEMS

Guido Agapito[1] [a], Giorgio Battistelli[2], Daniele Mari[2,3], Daniela Selvi[2] [b], Alberto Tesi[2], and Pietro Tesi[4,5]

[1] INAF - Osservatorio di Arcetri (INAF-Arcetri), Largo Enrico Fermi 5, 50125 Firenze, Italy
[2] University of Florence, Dipartimento di Ingegneria dell'Informazione (DINFO), Via di Santa Marta 3, 50139 Firenze, Italy
[3] Università della Calabria, Dipartimento di Ingegneria Informatica, Modellistica, Elettronica e Sistemistica (DIMES), Via P. Bucci, cubo 42/c, 87036 Arcavacata di Rende (CS), Italy
[4] University of Genoa, Dipartimento di Informatica, Bioingegneria, Robotica e Ingegneria dei Sistemi (DIBRIS), Via Opera Pia 13, 16145 Genova, Italy
[5] University of Groningen, Institute for Technology, Engineering & Management, Faculty of Mathematics and Natural Sciences, Nijenborgh 4, 9747 AG Groningen, The Netherlands

Abstract. The problem of reducing the effects of wavefront distortion and structural vibrations in ground-based telescopes is addressed within a modal-control framework. The proposed approach aims at optimizing the parameters of a given modal stabilizing controller with respect to a performance criterion which reflects the residual phase variance and is defined on a sampled frequency domain. This framework makes it possible to account for turbulence and vibration profiles of arbitrary complexity (even empirical power spectral densities from data), while the controller order can be kept at a desired value. Moreover it is possible to take into account additional requirements, as robustness in the presence of disturbances whose intensity and frequency profile vary with time. The proposed design procedure results in solving a minmax problem and can be converted into a linear programming problem with quadratic constraints, for which there exist several standard optimization techniques. The optimization starts from a given stabilizing controller which can be either a non-model-based controller (in this case no identification effort is required), or a model-based controller synthesized by means of turbulence and vibration models of limited complexity. In this sense the approach can be viewed not only as alternative, but also as cooperative with other control design approaches. The results obtained by means of an End-to-End simulator are shown to emphasize the power of the proposed method.

1 Introduction

Adaptive optics (AO) is a technique used to reduce the effects of wavefront distortion in ground-based telescopes caused by the atmospheric turbulence; in addition, it is known that AO performance can be reduced by structural vibrations arising, for example, in situations such as wind shaking and telescope orientation.

AO system design is a challenging problem from a control engineering perspective (see, for example, [1–4]). In this paper we deal with the AO system architecture adopted for the Large Binocular Telescope (LBT) [5, 6] and one of the two Magellan Telescopes [7], and which will be used also for the upgrading of one of the four Very Large Telescopes [8] and the Giant

---

[a] agapito@arcetri.astro.it
[b] daniela.selvi@unifi.it

Magellan Telescope [9]. The considered AO unit is made of a pyramid wavefront sensor (WFS), an adaptive secondary mirror (ASM), and a real-time computer (RTC).

Different approaches to AO control are possible, that is, those which do not rely on any identification of the involved disturbances, and those which adopt model-based design techniques by using models of turbulence and vibrations. Model-based approaches can ideally achieve the best performance, but they need a (sometimes not negligible) identification effort and provide controllers whose order is strictly related to the order of the models; as it is important that the models are sufficiently accurate at least in specific frequency ranges, high-order controllers can in general result from the synthesis procedure. Moreover, variations in the operating conditions would require to repeat the controller design procedure on the basis of models that better match the actual disturbance evolution.

The approach proposed in this work extends the results of [4], and aims at providing a control design technique which is optimal with respect to a certain performance criterion and proves robust with respect to variations in the operating conditions. Specifically, the technique operates in a modal control framework, and optimizes the parameters of a given modal stabilizing controller with respect to a performance criterion which reflects an upper-bound on the residual phase variance in the “worst case” and is defined on a sampled frequency domain. This means that only samples of turbulence and vibrations frequency profiles and, accordingly, samples of the measurement noise frequency profile are employed, rather than analytical models. This framework makes it possible to account for turbulence and vibration profiles of arbitrary complexity, even empirical power spectral densities (PSDs), while the controller order can be kept at a desired value. In addition, the optimization starts from a given controller, which can be either a non-model-based controller, or a model-based one synthesized on the basis of disturbance models of limited complexity. In this sense the approach can be viewed not only as alternative, but also as cooperative with other control design approaches.

The paper is organized as follows. Section 2 shows how variations in the operating conditions impact the disturbance frequency profiles, and describes the proposed approach. Simulation results carried out on an End-to-End simulator are shown in Section 3, while Section 4 provides concluding remarks.

2 Different operating conditions and design algorithm

It is important when dealing with disturbance attenuation problems to carefully account for variations in the operating conditions. For example, variations in the wind speed or in the seeing value have an impact on the disturbance frequency profile (see, for example, [10]).

To elaborate on this issue, consider a typical turbulence and vibration PSD profile related to tip as shown in Fig. 1. This profile has been obtained by considering the turbulent phase as represented by a set of two turbulent layers displaced at 15 and 18 m/s, respectively - mean wind speed 16.5 m/s - and producing a seeing value of 0.8”; as for the considered vibrations, they have amplitude of 20 mas and occur at 13 and 22 Hz, respectively.

According to the Taylor’s frozen flow hypothesis [11] we consider that variations of the wind speed do not affect the turbulence seeing; wind speed impacts the vibration amplitude and the turbulence cutoff frequency instead. In fact, an increase in the wind mean speed implies that the cutoff frequency moves towards higher frequencies, so that the whole turbulence profile moves towards higher frequencies. At the same time, if the wind speed corresponding to the ground layer increases, the structural vibrations increase their amplitude, due to the increased pres-

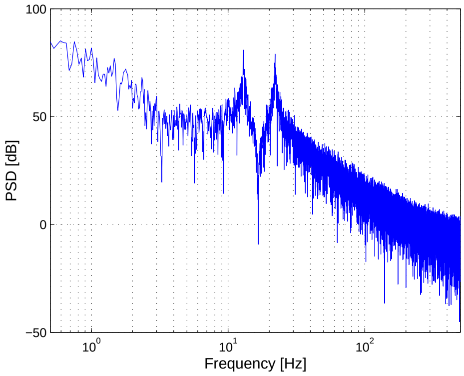

Fig. 1. Turbulence and vibration PSD related to tip.

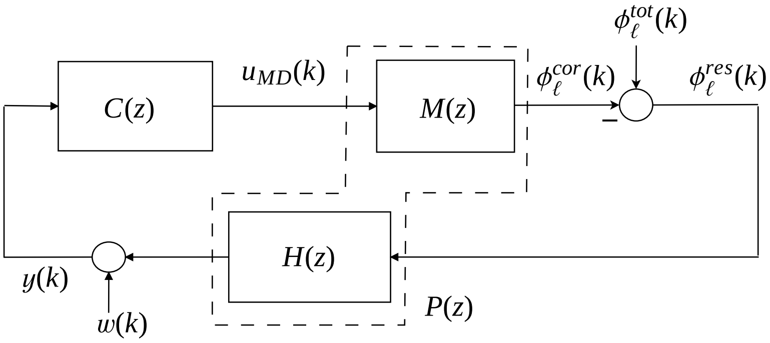

Fig. 2. Modal control scheme.

sure that the wind entering the dome exerts on the telescope structure. Similarly, a decreasing wind speed corresponds to the cutoff frequency (and so the whole turbulence frequency profile) moving towards lower frequencies, and to decreasing amplitude of the structural vibrations.

Variations of the seeing conditions act in turn on the turbulence profile. Specifically, for increasing seeing values the turbulence power increases, so that the profile rises (but the frequency range does not change); similarly, for decreasing seeing values the turbulence power decreases, so that the profile moves downwards.

In addition to the wind speed, also the wind direction affects the vibration amplitude. In fact structural vibrations arise partly due to the wind entering the dome. It is possible that the wind pressure greatly decreases when looking off-wind, because the dome shields the telescope structure. In this case some vibrations may disappear or at least become negligible.

The technique proposed in this paper extends the results provided in [4] to account for robustness under various operating conditions. The aim is to make the AO system able to regulate the residual phase about zero under every possible condition; this can be achieved by minimizing the sampled-valued variance of the residual phase $\phi_\ell^{res}(k)$

$$\lim_{h\to\infty} \frac{1}{h+1} \sum_{k=0}^{h} |\phi_\ell^{res}(k)|^2 \qquad (1)$$

in the “worst case” with respect to a certain number L of considered situations. The subscript ℓ, which refers to these operating conditions, ranges from 1 to L.

The AO control system architecture considered in this paper is the one described in the modal framework of [4]; further details can be found also in [5] and [6]. The control scheme is shown in Fig. 2, where $C(z)$ represents the transfer function of a dedicated controller synthesized for tip or tilt (all the other modes are controlled by a set of integrators), while $P(z)$ denotes the plant transfer function, consisting of the ensemble of the ASM and WFS dynamics (represented by $M(z)$ and $H(z)$, respectively, both assumed to behave as a unit delay). The residual phase $\phi_\ell^{res}(k)$ is computed as the difference between the phase aberration $\phi_\ell^{tot}(k)$, caused by turbulence and vibration, and the correction phase $\phi_\ell^{cor}(k)$. Finally, the measured output $y(k)$ is corrupted by a noise $w(k)$.

We recall that, according to the *Youla parametrization*, we can express the set of all the stabilizing controllers as (see [12])

$$\mathcal{C}(P) = \left\{ C(z) = \frac{Q(z)}{1 - P(z)\,Q(z)},\ \ Q(z) \in \mathcal{S} \right\}, \tag{2}$$

where $\mathcal{S}$ is the set of all the proper transfer functions with poles inside the open unit disk. The transfer function $Q(z)$ is known as the *Youla parameter*; since it is arbitrary (provided that it is stable), it can be tuned with the aim to achieve the desired control performance. A possible choice is to define the Youla parameter as a linear combination of stable functions by means of a parameter vector ρ, *i.e*

$$Q(z) = Q(z,\rho) = \psi^T(z)\rho, \tag{3}$$

where $\psi(z) = [\psi_1(z)\,\psi_2(z)\,\cdots\,\psi_n(z)]^T$ and $\rho = [\rho_1\,\rho_2\,\cdots\,\rho_n]^T$. Thanks to this definition, the tuning procedure can be performed with respect to ρ.

We address the problem as described below. We suppose that L estimates of the turbulence and vibration PSDs, denoted by $\hat{\Upsilon}_\ell^{\phi}(\omega_i)$, $\ell \in \{1,2,...,L\}$, related to tip and tilt, are available at ω_i, $i \in \{1,2,...,N\}$; moreover, we account for the contribution of the measurement noise $w(k)$ in the residual phase, and assume that an estimate of its PSD, denoted by $\hat{\Upsilon}^{w}(\omega_i)$, related to tip and tilt, is also available at ω_i, $i \in \{1,2,...,N\}$. As detailed in [4], by exploiting the Parseval's relationship and replacing the integral operator with a finite sum over the frequencies $\omega_1, \omega_2, ..., \omega_N$, the residual phase variance (1) can be expressed as

$$f(\rho,\ell) = \frac{1}{N}\sum_{i=1}^{N} \left\{ |(1 - P(e^{j\omega_i})\psi^T(e^{j\omega_i})\rho)|^2\, \hat{\Upsilon}_\ell^{\phi}(\omega_i) + |M(e^{j\omega_i})\psi^T(e^{j\omega_i})\rho|^2\, \hat{\Upsilon}^{w}(\omega_i) \right\}. \tag{4}$$

Then we can formulate the optimization problem to be solved:

$$\min_{\rho}\ \max_{\ell \in \{1,2,...,L\}} f(\rho,\ell) \tag{5}$$

s.t.

$$C(z,\rho) \in \mathcal{S}. \tag{6}$$

The constraint expressed by Eq. (6) requires that the controller $C(z)$, which, thanks to Eqs. (2) and (3), can be expressed as a function of the parameter vector ρ, is stable. This requirement is necessary to ensure the overall stability in circumstances that cause the command signal to be interrupted for a certain number of time steps (for example, when actuators are required to perform not-compatible actions, as force or stroke requirements which are out of range). We point out that the constraint expressed by Eq. (6) implies that the feasible set is in general non-convex. To overcome this drawback, an iterative design procedure can be performed, as suggested in [4]. The procedure starts from any stable and stabilizing controller $\hat{C}(z)$, which can be either a non-model-based controller, or a model-based one synthesized by means of disturbance models of limited complexity.

In practice the minmax problem shown above can be addressed by solving

$$\min_{\gamma,\rho} \gamma$$

s.t.

$$f(\rho, \ell) < \gamma\,, \quad \ell = 1, 2, ..., L$$
$$C(z, \rho) \in \mathcal{S}.$$

3 Simulation results

In this section we show simulation results carried out on an End-to-End simulator developed at the Arcetri Astrophysical Observatory. Most of the parameter values are related to the LBT AO control architecture and typical operating conditions, but we point out that the analysis is intended to be a preliminary study of the effectiveness of the technique for a generic telescope equipped with a Single Conjugate AO system.

In the simulations that follow, we assumed that the telescope diameter is 8.222 m with a central obstruction of 11%. The tests were carried out by setting a sampling frequency of 1000 Hz. The Strehl ratio (SR) was calculated at 1.65 μm (H band).

The pyramid WFS with tilt modulation was simulated with a full Fourier-optics code developed at the Arcetri Astrophysical Observatory.

The turbulent phase is represented by a set of two turbulent layers; each layer corresponds to a phase screen, which is generated following the McGlamery method [13]. They have altitude of 0 and 6000 m over the telescope and their relative intensity is 60% and 40%, respectively. The temporal evolution of the turbulence is simulated, based on the Taylor's hypothesis, by displacing the phase screens in front of the telescope pupil according to the user-specified speed. The considered outer scale is 40 m.

As for the ASM, in order to model its spatial response influence functions determined via the Finite Elements Analysis model of the LBT ASM are used. We use a set of theoretical tilt-free Karhunrn-Loève modes [14] combined with the Zernike Tip and Tilt to form an orthogonal set on the telescope pupil [15]. These modes are projected onto the ASM influence functions. The temporal response is considered as a delay of 1 ms.

Within the simulations described hereafter, we synthesized dedicated controllers for tip and tilt by initializing the proposed procedure from the non-model-based controller

$$\hat{C}(z) = \frac{0.65z}{z - 0.95}. \tag{7}$$

The PSDs of $\phi_\ell^{tot}(k)$ and the PSD of $w(k)$ were sampled by selecting $N = 500$ samples with linear gridding. Except for tip and tilt, all the modes were controlled by a set of integrators with optimized gains obtained by means of the Optimized Modal Gain Integrator (OMGI) approach [16].

We considered several possible scenarios. First of all, we assumed a particular condition as the "nominal" scenario (scenario A) and carried out some tests by varying the parameters related to the turbulence profile, that is, wind mean speed and seeing value. The considered scenarios are listed below:

A) wind mean speed of 16.5 m/s, seeing value of 0.8”, vibrations of 20 mas at 13 and 22 Hz;
B) wind mean speed of 16.5 m/s, seeing value of 0.6”, vibrations of 20 mas at 13 and 22 Hz;
C) wind mean speed of 16.5 m/s, seeing value of 1.0”, vibrations of 20 mas at 13 and 22 Hz;
D) wind mean speed of 12.4 m/s, seeing value of 0.8”, vibrations of 11 mas at 13 and 22 Hz;
E) wind mean speed of 20.6 m/s, seeing value of 0.8”, vibrations of 31 mas at 13 and 22 Hz.

The results, in terms of the achieved SR, obtained with the described technique from the simulation tests are compared with the ones obtained by using either $\hat{C}(z)$ as the dedicated controller for tip and tilt, or integrators with optimized gain (OMGI) for all the modes. We refer by C_{OMGI} to the OMGI controller (for all the modes); by $\hat{C}$ to the initial controller as the dedicated controller; and by C_{opt} to the optimized robust controller as the dedicated controller. The left column of Fig. 3 shows the Point Spread Function (PSF) profiles related to the proposed scenarios (from A to E) when using each of them.

Further, we compared the results obtained by using C_{OMGI}, $\hat{C}$ and C_{opt} when the telescope structure is affected by different vibrations. Specifically, we supposed that the vibration at 22 Hz becomes negligible when looking off-wind (scenario F); further, in addition to the aforementioned vibration at 13 and 22 Hz, we assumed that some equipment (subsystems such as cooling pumps, fans, or hydraulic system) being switched on give rise to another vibration, occurring at 30 Hz (scenario G). Summing up, we considered the following scenarios:

F) wind mean speed of 16.5 m/s, seeing value of 0.8”, vibration of 20 mas at 13 Hz;
A) wind mean speed of 16.5 m/s, seeing value of 0.8”, vibrations of 20 mas at 13 and 22 Hz;
G) wind mean speed of 16.5 m/s, seeing value of 0.8”, vibrations of 20 mas at 13, 22 and 30 Hz.

The right column of Fig. 3 shows the Point Spread Function (PSF) profiles related to the proposed scenarios F, A and G.

Table 1. Performance parameters related to C_{OMGI}, $\hat{C}$ and C_{opt}.

	C_{OMGI}	$\hat{C}$	C_{opt}
min SR (%)	69.35	40.49	79.72
mean SR (%)	78.86	57.77	86.91
STD	8.31	11.30	3.98

Table 1 shows, for each controller, the minimal value of the SR obtained within the overall considered scenarios (from A to G) and the corresponding mean value and standard deviation (STD). The robust controller synthesized according to the proposed algorithm is able to achieve the better performance in terms of the SR with the least standard deviation.

4 Concluding remarks

In this paper we proposed a control design approach aiming at optimizing the parameters of a modal controller with respect to a performance criterion which reflects an upper-bound on the residual phase variance in the “worst case” among different operating conditions. The optimization technique is able to use disturbance frequency profiles of arbitrary complexity and even empirical PSDs, while the controller order can be kept at a desired value. The approach results in solving a minmax problem and can be converted into a linear programming problem

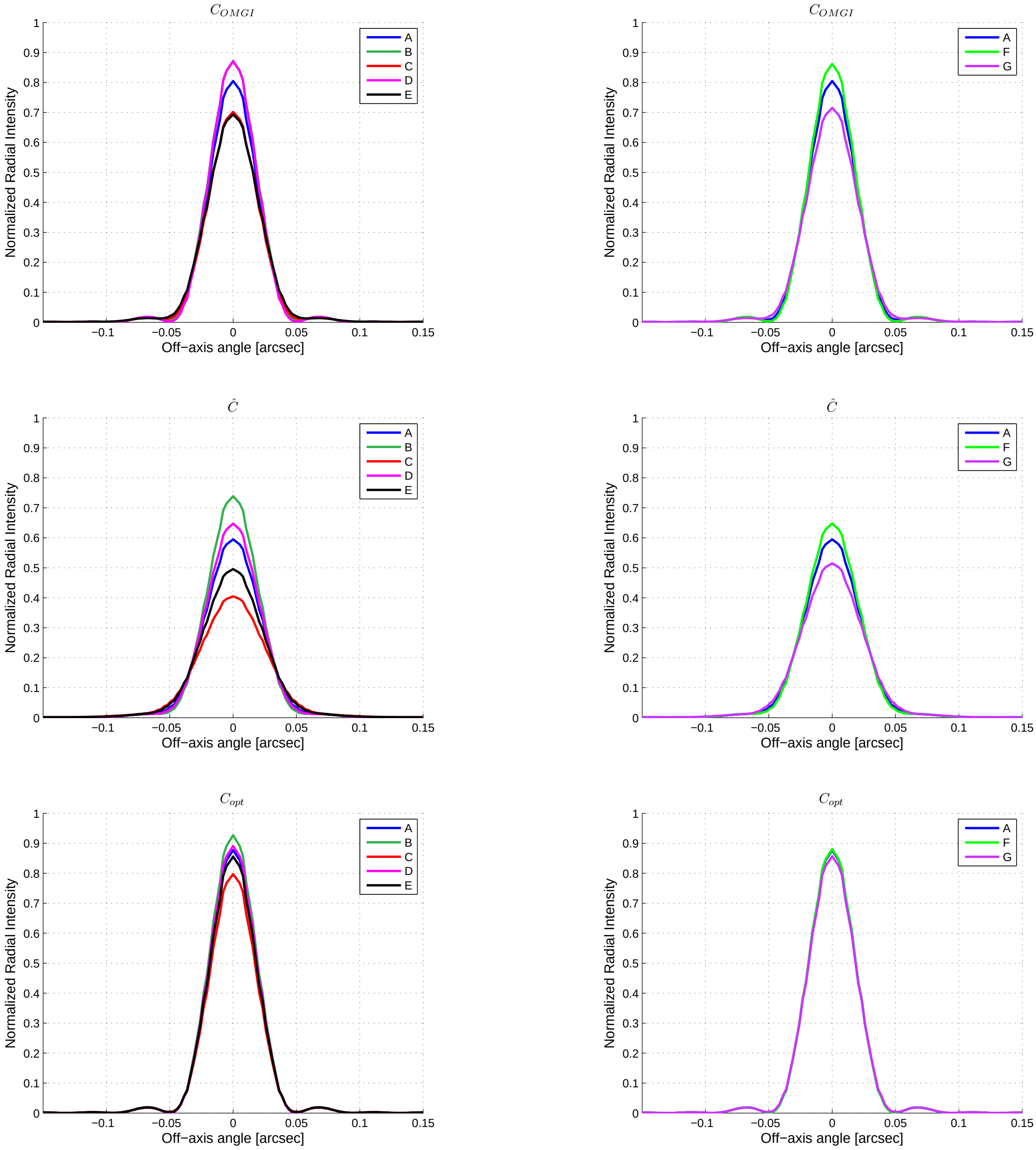

Fig. 3. PSF profiles resulting from the use of the considered controllers in the control loop. Left column: scenarios from A to E. Right column: scenarios F, A and G.

with quadratic constraints, for which there exist several standard optimization techniques. The algorithm can be initialized from either a non-model-based controller or a model-based one. Simulation results obtained on an End-to-End simulator developed at the Arcetri Astrophysical Observatory show that the design technique provides controllers which are able to achieve high performance levels in terms of the SR with slight differences in the SR value among the various operating conditions which have been considered.

References

1. C. Kulcsár, H.-F. Raynaud, C. Petit, J.-M. Conan, and P. Viaris de Lesegno, *Optimal control, observers and integrators in adaptive optics,* Optics Express **14**(17), (2006) 7464–7476.
2. C. Petit, J.-M. Conan, C. Kulcsár, H.-F. Raynaud, and T. Fusco, *First laboratory validation of vibration filtering with LQG control law for adaptive optics,* Optics Express **16**(1), (2008) 87–97.
3. *Adaptive optics control for ground based telescopes,* in *Special Issue of the European Journal of Control*, *Vol*. 17, *No*. 3, J.-M. Conan, C.Kulcsár, and H.-F. Raynaud, eds. (Lavoisier, 2011).
4. G. Agapito, G. Battistelli, D. Mari, D. Selvi, A. Tesi, and P. Tesi, *Frequency based design of modal controllers for adaptive optics systems,* Optics Express **20**(24), (2012) 27108–27122.
5. G. Agapito, F. Quirós-Pacheco, P. Tesi, A. Riccardi, and S. Esposito, *Observer-based control techniques for the LBT adaptive optics under telescope vibrations,* European Journal of Control **17**(3), (2011) 316–326.
6. G. Agapito, S.Baldi, G.Battistelli, D.Mari, E.Mosca, and A.Riccardi, *Automatic tuning of the internal position control of an adaptive secondary mirror,* European Journal of Control **17**(3), (2011) 273–289.
7. L. M. Close, J. R. Males, D. A. Kopon, V. Gasho, K. B. Follette, P. Hinz, K. Morzinski, A. Uomoto, T. Hare, A. Riccardi, S. Esposito, A. Puglisi, E. Pinna, L. Busoni, C. Arcidiacono, M. Xompero, R. Briguglio, F. Quirós-Pacheco and J. Argomedo, *First closed-loop visible AO test results for the advanced adaptive secondary AO system for the Magellan Telescope: MagAO's performance and status,* Proceedings of SPIE **8447**, Adaptive Optics Systems III, (2012) 84470X-1 – 84470X-16.
8. P. Amico, E. Marchetti, F. Pedichini, A. Baruffolo, B. Delabre, M. Duchateau, M. Ekinci, D. Fantinel, E. Fedrigo, G. Finger, C. Frank, R. Hofmann, P. Jolley, J. L. Lizon, M. Le Louarn, P.-Y. Madec, C. Soenke and H. Weisz, *The design of ERIS for the VLT,* Proceedings of SPIE **8446**, Ground-based and Airborne Instrumentation for Astronomy IV, (2012) 844620-1 – 844620-17.
9. A. H. Bouchez, D. Scott Acton, G. Agapito, C. Arcidiacono, F. Bennet, V. Biliotti, M. Bonaglia, R. Briguglio, G. Brusa-Zappellini, L. Busoni, L. Carbonaro, J. L. Codona, R. Conan, T. Connors, O. Durney, B. Espeland, S. Esposito, L. Fini, R. Gardhouse, T. M. Gauron, M. Hart, P. M. Hinz, S. Kanneganti, E. J. Kibblewhite, R. P. Knox, B. A. McLeod, T. McMahon, M. Montoya, T. J. Norton, M. P. Ordway, C. d'Orgeville, S. Parcell, P. K. Piatrou, E. Pinna, I. Price, A. Puglisi, F. Quirós-Pacheco, A. Riccardi, J. B. Roll, G. Trancho, K. Uhlendorf, V. Vaitheeswaran, M. A. van Dam, D. Weaver and M. Xompero, *The Giant Magellan Telescope Adaptive Optics Program,* Proceedings of SPIE **8447**, Adaptive Optics Systems III, (2012) 84471I-1 – 84471I-12.
10. J.-M. Conan, G. Rousset and P.-Y. Madec, *Wave-front temporal spectra in high-resolution imaging through turbulence,* Journal of the Optical Society of America A **12**, (1995) 1559–1570.
11. G. I. Taylor, *The spectrum of turbulence,* Proceedings of the Royal Society of London Series A, **164**, (1938) 476–490.
12. J. Doyle, B. Francis, and A. Tannenbaum, *Feedback Control Theory* (Macmillan Publishing Company, 1992), 63–85.
13. B. L. McGlamery, *Computer simulation studies of compensation of turbulence degraded images,* Proceedings of SPIE, **74**(9), (1976) 225–233.
14. J. Y. Wang and J. K. Markey, *Modal compensation of atmospheric turbulence phase distortion,* Journal of the Optical Society of America **68**, (1978) 78–87.
15. F. Quirós-Pacheco, L. Busoni, G. Agapito, S. Esposito, E. Pinna, A. Puglisi and A. Riccardi, *First light AO (FLAO) system for LBT: performance analysis and optimization,* Proceedings of SPIE **7736**, (2010) 77363H-1 – 77363H-10.
16. C. Dessenne, P.-Y. Madec, and G. Rousset, *Optimization of a predictive controller for closed-loop adaptive optics,* Applied Optics, **37**(21), (1998) 4623–4633.