DEVELOPMENT OF A PYRAMIDAL WAVEFRONT SENSOR TEST-BENCH AT INO

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Abstract

The key technical element of the adaptive optics (AO) in astronomy is the wavefront sensing (WFS). One of the advantages of the pyramid wavefront sensor (P-WFS) over the widely used Shack-Hartmann wavefront sensor seems to be the increased sensitivity in closed-loop applications. A high-sensitivity and large dynamic-range WFS, such as P-WFS technology, still needs to be further investigated for proper justification in future Extremely Large Telescopes (ELT) application. At INO, we have recently carried out the optical design, testing and performance evaluation of a P-WFS bench setup.

The optical design of the bench setup mainly consists of the super-LED fiber source, source collimator, spatial light modulator (SLM), relay lenses, tip-tilt mirror, Fourier-transforming lens, and a four-faceted glass pyramid with a large vertex angle as well as pupil re-imaged optics. The phase-only SLM has been introduced in the bench setup to generate atmospheric turbulence with a maximum phase shift of more than 2π at each pixel (256 grey levels). Like a modified Foucault knife-edge test, the refractive pyramid element is used to produce four images of the entrance pupil on a CCD camera. The Fourier-transforming lens, which is used before the pyramid prism, is designed for telecentric output to allow dynamic modulation (rotation of the beam around the pyramid-prism center) from a tip-tilt mirror.

Furthermore, a P-WFS diffraction-based model has been developed. This model includes most of the system limitations such as the SLM discrete voltage steps and the CCD pixel pitch. The pyramid effects (edges and tip) are considered as well. Finally, the phase-reconstruction errors of the P-WFS test bench have been compared with success to those from the diffraction-based model and also to those of a Shack-Hartmann, showing the regions of interest of the former system. The bench setup was thereafter lend to HIA for further testing.

1. Introduction

The pyramidal wavefront sensor (P-WFS) concept was first developed by Ragazzoni [1] for astronomical applications. A refractive element (the pyramid) is used to produce four images of the entrance pupil, which are thereafter combined in post-processing to retrieve the incident wavefront. For
Adaptive Optics systems in closed-loop regime an advantage often cited of P-WFS over the widely used Shack-Hartmann wavefront sensor (SH-WFS) is the increased sensitivity [2]. Unlike the SH-WFS, which uses small-aperture micro-lenses to sample the telescope pupil, a P-WFS is only limited by diffraction effects introduced by the whole telescope aperture. It offers more range in limiting magnitude because it is less affected by aliasing at the bright end and it is more sensitive at the faint end than a SH-WFS. The sensitivity is also adjustable to source brightness through binning of the four images (at the expense of the wavefront sampling) whereas in the case of a SH-WFS the sub-apertures are not adjustable in size. Furthermore, the P-WFS dynamic range is adjustable through the amplitude of the pyramid spatial modulation.

On the other hand, the need for dynamic modulation can be a disadvantage because it requires a moving component at high frequency inside the instrument. Also the very nature of the pyramid prism can introduce chromatic aberrations and limit the wavelength spectral range of the WFS.

The P-WFS technology still needs to be further investigated for proper justification in future Extremely Large Telescopes (ELT) application. At INO, we have recently carried out the optical design, testing and performance evaluation of a P-WFS bench setup. We present hereafter the results of preliminary tests carried out on the first iteration of a demonstrator.

2. P-WFS principle of operation

2.1. P-WFS measurement principle

The operation principle of a P-WFS is illustrated in Figure 1. The field incident on the pupil (denoted by \( U_0 \)) is focalised on the tip of a pyramid by a lens \( L_1 \) and is split into four beam by the four facets. Four conjugated images of the pupil are then produced on CCD by the second lens \( L_2 \). One important feature of the P-WFS is the adjustable gain, which is possible by a modulation of the pyramid (i.e a periodic displacement of the pyramid in the \( L_1 \) focal plane). Alternatively, the modulation can be achieved by the use of a tip-tilt mirror located in the pupil plane. The tip-tilt is rotated about the mirror central axis, which produce telecentric output. This feature, combined with the CCD being located in the lens focal plane, produces correctly registered pupils, i.e. they are images on the same CCD pixel whatever the tip-tilt angle. The aim of the P-WFS is to evaluate the wavefront \( W \) of the incident field \( U_0 \):

\[
U_0(x_0, y_0) = A(x_0, y_0) \exp \left( \frac{2\pi}{\lambda} W(x_0, y_0) \right)
\]

where \( A \) is the field amplitude. From diffraction theory, the intensity in the CCD plane can evaluated according to:

\[
I_i(x_i, y_i, t) \propto \left| U_i(x_i, y_i, t) \right|^2
\]

\[
U_i(x_i, y_i, t) = -\frac{e^{i2k(f_1 + f_2)}}{\lambda^2 f_1 f_2} \iint dx_p dy_p H(x_p, y_p, t) e^{-i(k(x_p + y_p)/f_2)} \iint dx_0 dy_0 U_0(x_0, y_0) e^{-i(k(x_0 + y_0)/f_1)}
\]

The time dependence is introduced into the pyramid phase transformation \( H(x_p, y_p, t) \) due to the time-varying position of the pyramid relatively to the focused beam.
The P-WFS signals are defined by:

\[
S_x(x,y) = \langle I_1(x,y) \rangle + \langle I_4(x,y) \rangle - \langle I_2(x,y) \rangle - \langle I_3(x,y) \rangle \\
S_y(x,y) = \langle I_1(x,y) \rangle + \langle I_2(x,y) \rangle - \langle I_3(x,y) \rangle - \langle I_4(x,y) \rangle
\]  

(3)

where the pupil system of references are illustrated in Figure 2. The coordinates \((x,y)\) refers to co-registered pixels in the four pupil images and the \(< >\) brackets indicate that the pupil signals are averaged over a modulation period.

\[
W_m(x_0, y_0) = \sum_{k=1}^{N_c} c_k Z_k(\frac{x_0}{R}, \frac{y_0}{R})
\]  

(4)

where \(R\) is the wavefront radius, \(Z_k\) is the \(k\)th Zernike polynomial (described by the integer parameters \(n_k\) and \(m_k\) with \(n_k \geq |m_k|\)), \(c_k\) its amplitude and \(N_c\) the number of terms in the expansion. The Zernike polynomial indices were ordered according to the aberration order \((n_k + |m_k| - 1)\). It can be shown from diffraction theory [4], in the limit of small aberrations \((W_m/\lambda \ll 1)\), that

\[
S_x(x,y,\alpha) = S_{x0}(x,y,\alpha) + \sum_{k=1}^{N_c} c_k T_{xk}(x,y,\alpha) \\
S_y(x,y,\alpha) = S_{y0}(x,y,\alpha) + \sum_{k=1}^{N_c} c_k T_{yk}(x,y,\alpha)
\]  

(5)
where the interaction matrices \( T_{xk} \) and \( T_{yk} \) are functions of the \( k \)th Zernike polynomial. The P-WFS signals are therefore linear (only in the small aberration limit) with the polynomial indices. \( S_{x0} \) and \( S_{y0} \) represent offsets and \( \alpha \) is the modulation amplitude. The interaction matrices and the offsets are characterized for each modulation amplitude. In matrix form, we have that \( S = TC \), where \( S = [S_x, S_{x0}, S_y, S_{y0}]^T \), \( t \) being the transpose operator, \( T \) is composed of \( T_{xk} \) and \( T_{yk} \) and \( C \) is the Zernike coefficient vector. The combined response matrix \( T \) can be decomposed using Singular Value Decomposition (SVD)

\[
T = \Omega \Sigma V^T
\]

where \( \Omega \Omega = I \), \( \Sigma \) is a diagonal matrix and \( VV = 1 \). Finally, the Zernike coefficient vector corresponding to a measured P-WFS signal \( S_{test} \) will be given by:

\[
C_{test} = V\Sigma^{-1}\Omega^T S_{test}
\]

### 3. The INO P-WFS test bench

The set-up built at INO is basically composed of two independent sub-assemblies: a waveform generator and calibrator and the proper pyramid wavefront sensor. A general view of the setup is seen in Figure 2.

![Figure 2: General view of the setup at INO.](image)

The light source is a 790nm fibre-pigtailed Superluminescent LED (S-LED) whose low-coherence \(~30\)nm spectral width limits risks of speckle.
After collimation the beam is reflected onto a phase-only LCD Spatial Light Modulator (SLM). The SLM receives electrical signals from a PC video card controlled by a Matlab interface. In this way arbitrary waveforms can be generated. A pupil stop (3 mm radius) is set in front of the SLM active area.

A commercial Shack-Hartmann analyser allows to check in real time the exact waveform generated by the SLM. It is also used to calibrate the SLM and the overall beam train.

The optical architecture of the main part of the demonstrator was optimized in order to respect the fundamental design characteristic of a pyramid wavefront sensor the sense that the tip-tilt (modulation) mirror is conjugated to the exit pupil of the system under test.

The vertex of the pyramid prism is located at the focal plane of the system. The large f/ number (f#=45) produces a spot size of 85 µm at the tip of the pyramid, well suited to the prism geometry. A relay lens situated after the pyramid forms on the CCD the images of the four sub-pupils generated by the faces of the pyramid. The CCD pixels are binned to obtain about 32 binned pixels on the diameter of each sub-pupil image. The centers of two adjacent sub-pupil images are separated by 60 binned pixels. The system is telecentric so that when the mirror is modulated the sub-pupil images do not move on the CCD plane.

All lenses in the system were chosen to be off-the-shelf components in order to minimize the overall cost and delivery time but still achieve excellent image quality, better than $\lambda/24$ with beam scanning.

4. Preliminary test results

4.1. Test at INO

The signals measured with the P-WFS test bench were validated with a diffraction-based model (see Eq. (2)) incorporating most of the system specifications: more particularly the pyramid specifications. The parameters defining the pyramid are the facet angle ($\theta=3.379^\circ$), the edge ($e \sim 6 \mu m$) and tip ($L_1 \sim 50 \mu m, L_2 \sim 30 \mu m$), as shown in Figure 4.

![Figure 4: The pyramid seen from above. Left panel: on the microscope; right panel: an illustration.](image)

The pyramid tip being not-symmetric, the energy transferred from the beam to the faces 1 and 3 is therefore greater than for faces 2 and 4. This imbalance in the energy transfer is transposed to the intensity of the pupil images, as shown in Figure 5 for data (left panels) and simulations (right panels). Using the definition in Figure 2, $I_1$ and $I_3$ are brighter than $I_2$ and $I_4$. The images shown in Figure 5 correspond to the no-modulation case ($\alpha=0$) for respectively no-aberration, defocus and $3^{rd}$ order spherical. A generally good agreement is found between data and simulation. The slight discrepancies might be due to a wavefront offset caused by an imperfect characterization of the SLM baseline.

The second validation/characterization of the P-WFS test bench was done by testing the wavefront reconstruction errors. These were validated with the diffraction-based model and the Shack-Hartmann.
Different wavefronts were randomly generated (the Zernike coefficients being weighted by a Kolmogoroff distribution [5]) for the case with \((\alpha=5\lambda/D)\) and without \((\alpha=0)\) modulation. The input phase variance and the reconstruction error variance are defined by

\[
\sigma^2_{in} = \frac{2\pi^2}{\lambda^2} \sum_{k=3}^{N_z} c_k^2 \delta_{0,m_k+1} \left( n_k + 1 \right)
\]

\[
\sigma^2_{error} = \frac{2\pi^2}{\lambda^2} \sum_{k=3}^{N_z} (c_k - \tilde{c}_k)^2 \delta_{0,m_k+1} \left( n_k + 1 \right)
\]

where \(c_k\) are the input Zernike coefficients and \(\tilde{c}_k\) are those reconstructed from Eq. (7). The tilts \((k=1, 2)\) aberrations were not included in the analysis. The test were done at \(\lambda=790\) nm and the beam was attenuated to about 1.5 nW input power on the CCD.

The results for \(N_z=35\) (9th order aberration) and \(N_z=80\) (15th order aberration) are shown in Figure 6. Each data point is obtained from an average over five different acquisitions, 30 snapshot per acquisition with 16.67 ms exposure time per snapshot. For both \(N_z=35\) and \(N_z=80\), the best reconstruction results were obtained without modulation at low input phase variance, in agreement with the simulation. The input and reconstructed Zernike coefficients are compared in the right panel of Figure 6 for no-modulation and \(N_z=35\), showing the high performance of the sensor in that region. The modulation provides a (small) reduction of the reconstruction error at larger input phase variance, relatively to the no-modulation case. Despite the smaller reconstruction error of the Shack-Hartmann at high input phase variance, the overall reconstruction error of a sensor in a close-loop adaptive optics is defined by the low input phase variance region. Therefore, according to our results, and in agreement with Ref. [2], the P-WFS appears to have the better reconstruction error, at least for the no-modulation case.

The dominant source of noise in the test-bench originated from the CCD. Indeed, the output air flow from the CCD introduced vibrations in the system and particularly on the pyramid. These vibrations produced non-controlled modulations of the pyramid and caused wavefront reconstruction errors. The

![Figure 5: Pupil images from data and simulations for the no-modulation case. Top-panel: no wavefront aberration; middle panel: defocus; bottom panel: 3rd order spherical.](image)
noise amplitude was evaluated from the simulation shown in Figure 7. Accordingly, the noise would have caused a degradation of the reconstruction error variance by a factor ~ 3 at low input phase variance (0.01 rad²), without perturbing the reconstruction at higher input phase variance (100 rad²).

Furthermore, the limited accuracy of the tip-tilt motorized actuators caused additional reconstruction errors for the cases with modulation. The inaccuracy of these actuators and their effect on the modulation were studied further by collaborators at the HIA. This is the subject of the next section.

Figure 6: Left panel: wavefront reconstruction error variance for P-WFS (data and simulation) and Shack-Hartmann. The incident and reconstructed wavefronts are based on 35 (top panel) and 80 (bottom panel) Zernike coefficients. Right panel: P-WFS input and output Zernike coefficients for an input variance of 0.01 rad² (35 Zernike coefficients). Each data point corresponds to 30 images co-added (16.67 ms exposure time per image) for an average over five different acquisitions.

Figure 7: Effect of noise for 0 λ/D (simulation).

4.2. Test at HIA

In order to study the effect of actuators on the modulation, a camera was inserted in the setup, just before the pyramid. The use of low cost actuators (Thorlabs Z806) with significant backlash (~ 5 μm)
reduced the reproducibility of periodic modulations. Preliminary test showed that for large modulations, a circular displacement was maintained. However, the center of this modulation appeared to drift. This was especially notable for smaller modulations, as shown in Figure 8. The “circles” show the variation the position of each point in the circle, indicating the erratic movement of the center of the beam while tracing the circle. This irregular tracing was also more notable for smaller modulations. These test suggested that these actuators were insufficient for any long-term measurement in real-time.

![Figure 8: Comparison between beam position for initial modulation and final modulation (100 times) for a) 1.4 λ/d and b) 11 λ/d.](image)

5. Conclusions
The first iteration of a Pyramid-WFS demonstrator was built at INO in order to acquire experience with the technology. Preliminary tests were carried out at INO and then at NRC-HIA Victoria. The tests with the system were done in open loop, static mode and with modulations at low temporal frequency. These tests showed that the wavefront reconstruction performances agree with simulations. The current limitations – mainly due to low-cost actuators for the modulation mirror, chosen for reasons of convenience – are well understood.

We are now planning the next phase of the project, for which the efforts will be directed towards the improvement of the system temporal and spectral band-passes. This will be done by the use of a piezo-electric modulation mirror, a polychromatic re-design of the optical beam train, a low-noise fast acquisition CCD camera, and the development of non-linear reconstruction algorithm.

The ultimate goal pursued at INO is to build a compact and modular P-WFS subsystem that will allow closing the loop on a deformable mirror and performing real life tests on an astronomical AO system like the one currently developed for the Mont-Megantic Observatory.

6. References