Third AO4ELT Conference - Adaptive Optics for Extremely Large Telescopes
Florence, Italy. May 2013
ISBN: 978-88-908876-0-4
DOI: 10.12839/AO4ELT3.13356

DEVELOPING PERFORMANCE ESTIMATES FOR HIGH PRECISION ASTROMETRY WITH TMT

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Abstract. Adaptive optics on Extremely Large Telescopes will open up many new science cases or expand existing science into regimes unattainable with the current generation of telescopes. One example of this is high-precision astrometry, which has requirements in the range from 10 to 50 micro-arcsec for some instruments and science cases. Achieving these requirements imposes stringent constraints on the design of the entire observatory, but also on the calibration procedures, observing sequences and the data analysis techniques.

This paper summarizes our efforts to develop a top down astrometry error budget for TMT. It is predominantly developed for the first-light AO system, NFIRAOS, and the IRIS instrument, but many terms are applicable to other configurations as well. Astrometry error sources are divided into 5 categories: reference source and catalog errors, atmospheric refraction correction errors, other residual atmospheric effects, opto-mechanical errors and focal plane measurement errors.

Results are developed in parametric form whenever possible. However, almost every error term in the error budget depends on the details of the astrometry observations, such as whether absolute or differential astrometry is the goal, whether one observes a sparse or crowded field, what the time scales of interest are, etc. Thus, it is not possible to develop a single error budget that applies to all science cases and separate budgets are developed and detailed for key astrometric observations.

Our error budget is consistent with the requirements for differential astrometry of tens of micro-arcsec for certain science cases. While no show stoppers have been found, the work has resulted in several modifications to the NFIRAOS optical surface specifications and reference source design that will help improve the achievable astrometry precision even further.

1 Astrometry with TMT

The future generation of extremely large telescopes (ELTs) will allow us to study the universe with a depth and precision previously not possible. For the Thirty Meter Telescope (TMT), high precision astrometry, that is, measuring the exact positions of the science objects, is one of the very important science capabilities and therefore one of the main drivers for its technical requirements. As an example, NFIRAOS, TMT's first-light facility adaptive optics (AO) system is required to produce 50 micro-arcsec differential astrometry in 100-second exposures over a 30 arcsec field of view in the H Band. This error is supposed to fall as $T^{-1/2}$ to a systematic floor of 10 micro-arcsec, where T is the integration time. Thus, errors need to be controlled to two to three orders of magnitude smaller than the diffraction limit. In relative terms, this is comparable

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to what can be done at current telescopes which reach levels of 100—300 micro-arcsec (and even below that in special cases), a similar factor below their diffraction limit (see, for example, Refs. [1–4]). In absolute terms, it means a significant step in control and characterization of error sources such as distortions to a level that has not yet been demonstrated in practice.

We are therefore working on a detailed analysis of the potential sources of errors and uncertainties of TMT astrometry observations, following in parts an analysis done for the E-ELT [5] and expanding upon it. This is done through a combination of observations at existing observatories, analytical calculations and simulations of the science fields, atmosphere and observatory. The analysis is not restricted to the case of differential astrometry in fields with many sources, which is the science case targeted by the requirement above, but is supposed to cover the entire range of astrometric science cases TMT is likely to be used for.

When trying to establish such an error budget, one soon discovers that it is not possible to come up with an error budget that covers all astrometric science cases by means of a few adjustable parameters, as the errors depend on the details of the astrometric observation not only quantitatively, but also qualitatively. The TMT astrometry error budget therefore consists of a number of ”generic” science cases which are treated parametrically, as well as of several specific science cases of particular interest that are analyzed individually, such as astrometric measurements of stellar orbits around the black hole at the center of the Milky Way. [6] This paper provides a summary of the error budget for the generic science cases.

2 TMT Astrometry Error Budget

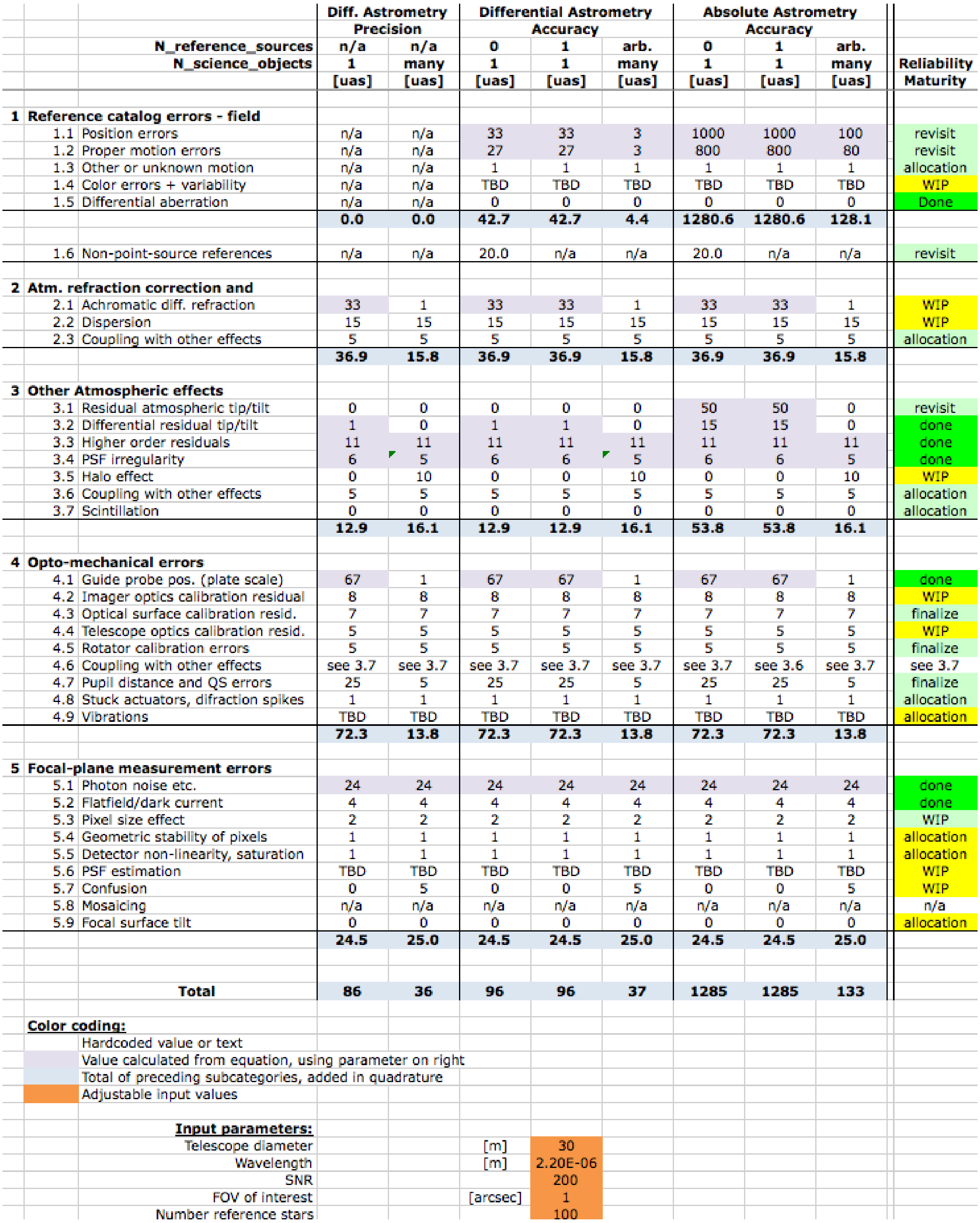
Figure 1 shows the individual terms of the TMT astrometry error budget for a total of eight generic science cases in an attempt to describe ”standard” or ”representative” observations in parametric form. Note that this is work in progress and that values are very dependent on the exact type of observation, even within the same category. At this time, this table should only be used for getting an impression of the dominant error terms and their general magnitudes. The last column also shows our estimate of the status and maturity of the respective error terms.

The generic science cases are characterized by two parameters. N_{ref} is the number of reference sources with known coordinates that are available from other observations (e.g. the HIPPARCOS or Gaia catalogs, or the masers around the Galactic Center) in the science field. $N_{ref} = 0$ means that no reference sources are available in the field, in which case the three NFIRAOS natural guide stars (NGSs) must be used as absolute reference sources.

$N_{science}$ is the number of science objects that can be used for distortion correction in post-processing. Thus, only objects in the field with relatively stable relative positions (or at least known relative proper motions) can be used for this. As a result, the case of determining the relative positions of the two stars in a binary system is equivalent to $N_{science} = 1$ even though there are two objects in the field. The same is true for the case of a number of fast-moving ejecta in a jet, independent of the number of resolved sources (unless their relative motions can be calibrated with high precision).

2.1 Differential astrometry precision

In the figure, differential astrometry precision refers to the case when only relative distances between objects are of interest and, moreover, those distances do not have to be calibrated in absolute terms. This case is, generally, not scientifically useful as absolute distances are needed



		Diff. Astrometry Precision		Differential Astrometry Accuracy			Absolute Astrometry Accuracy			
	N_reference_sources	n/a	n/a	0	1	arb.	0	1	arb.	
	N_science_objects	1	many	1	1	many	1	1	many	Reliability Maturity
		[uas]	[uas]	[uas]	[uas]	[uas]	[uas]	[uas]	[uas]	
1	**Reference catalog errors - field**									
1.1	Position errors	n/a	n/a	33	33	3	1000	1000	100	revisit
1.2	Proper motion errors	n/a	n/a	27	27	3	800	800	80	revisit
1.3	Other or unknown motion	n/a	n/a	1	1	1	1	1	1	allocation
1.4	Color errors + variability	n/a	n/a	TBD	TBD	TBD	TBD	TBD	TBD	WIP
1.5	Differential aberration	n/a	n/a	0	0	0	0	0	0	Done
		0.0	**0.0**	**42.7**	**42.7**	**4.4**	**1280.6**	**1280.6**	**128.1**	
1.6	Non-point-source references	n/a	n/a	20.0	n/a	n/a	20.0	n/a	n/a	revisit
2	**Atm. refraction correction and**									
2.1	Achromatic diff. refraction	33	1	33	33	1	33	33	1	WIP
2.2	Dispersion	15	15	15	15	15	15	15	15	WIP
2.3	Coupling with other effects	5	5	5	5	5	5	5	5	allocation
		36.9	**15.8**	**36.9**	**36.9**	**15.8**	**36.9**	**36.9**	**15.8**	
3	**Other Atmospheric effects**									
3.1	Residual atmospheric tip/tilt	0	0	0	0	0	50	50	0	revisit
3.2	Differential residual tip/tilt	1	0	1	1	0	15	15	0	done
3.3	Higher order residuals	11	11	11	11	11	11	11	11	done
3.4	PSF irregularity	6	5	6	6	5	6	6	5	done
3.5	Halo effect	0	10	0	0	10	0	0	10	WIP
3.6	Coupling with other effects	5	5	5	5	5	5	5	5	allocation
3.7	Scintillation	0	0	0	0	0	0	0	0	allocation
		12.9	**16.1**	**12.9**	**12.9**	**16.1**	**53.8**	**53.8**	**16.1**	
4	**Opto-mechanical errors**									
4.1	Guide probe pos. (plate scale)	67	1	67	67	1	67	67	1	done
4.2	Imager optics calibration residual	8	8	8	8	8	8	8	8	WIP
4.3	Optical surface calibration resid.	7	7	7	7	7	7	7	7	finalize
4.4	Telescope optics calibration resid.	5	5	5	5	5	5	5	5	WIP
4.5	Rotator calibration errors	5	5	5	5	5	5	5	5	finalize
4.6	Coupling with other effects	see 3.7	see 3.7	see 3.7	see 3.7	see 3.7	see 3.7	see 3.6	see 3.7	see 3.7
4.7	Pupil distance and QS errors	25	5	25	25	5	25	25	5	finalize
4.8	Stuck actuators, difraction spikes	1	1	1	1	1	1	1	1	allocation
4.9	Vibrations	TBD	TBD	TBD	TBD	TBD	TBD	TBD	TBD	allocation
		72.3	**13.8**	**72.3**	**72.3**	**13.8**	**72.3**	**72.3**	**13.8**	
5	**Focal-plane measurement errors**									
5.1	Photon noise etc.	24	24	24	24	24	24	24	24	done
5.2	Flatfield/dark current	4	4	4	4	4	4	4	4	done
5.3	Pixel size effect	2	2	2	2	2	2	2	2	WIP
5.4	Geometric stability of pixels	1	1	1	1	1	1	1	1	allocation
5.5	Detector non-linearity, saturation	1	1	1	1	1	1	1	1	allocation
5.6	PSF estimation	TBD	TBD	TBD	TBD	TBD	TBD	TBD	TBD	WIP
5.7	Confusion	0	5	0	0	5	0	0	5	WIP
5.8	Mosaicing	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a	n/a
5.9	Focal surface tilt	0	0	0	0	0	0	0	0	allocation
		24.5	**25.0**	**24.5**	**24.5**	**25.0**	**24.5**	**24.5**	**25.0**	
	Total	**86**	**36**	**96**	**96**	**37**	**1285**	**1285**	**133**	

Color coding:

⬜	Hardcoded value or text
🟪	Value calculated from equation, using parameter on right
🟦	Total of preceding subcategories, added in quadrature
🟧	Adjustable input values

Input parameters:

Telescope diameter	[m]	30
Wavelength	[m]	2.20E-06
SNR		200
FOV of interest	[arcsec]	1
Number reference stars		100

Fig. 1. Examples of possible values for the TMT astrometry error budget generic science cases. See text for details. Note that this is work in progress and that values are very dependent on the exact type of observation, even within the same category. At this time, this table should only be used for getting an impression of the dominant error terms and their general magnitudes.

for most, if not all, important TMT science cases, but it is included here to demonstrate the difference between differential astrometry precision and accuracy. This can then be used to clarify the required types of observations and data reduction procedures for a given science case.

The number of reference sources, N_{ref}, is irrelevant here as only relative distance are of interest. Two cases of number of science objects, $N_{science}$ are considered:

$\mathbf{N_{science}} \leq \mathbf{2}$: This means that at most tip/tilt can be removed (which is irrelevant to this case; this is therefore equivalent to the case of no a-posteriori distortion correction). Note that, as stated above, this is also representative of science cases with more than two objects that move relative to each other in an unknown fashion.

Large **$\mathbf{N_{science}}$** : Higher order distortions can be removed. For the most part, we only consider the removal of tip/tilt and plate-scale modes, which can be done with as few as three stars. However, the use of many stars means that these modes can be measured with high S/N and reduced aliasing of higher-order distortion modes. We therefore usually assume that they can be removed without residual, although exceptions to that rule might be considered in some cases.

2.2 Differential astrometry accuracy

Differential astrometry accuracy also deals with the relative distances between the science objects, but now they are required in units of physical sky coordinates. Both the number of science objects, $N_{science}$ and the number of reference sources, N_{ref}, matter in this case. The same values of $N_{science}$ as in the previous section are considered. Meaningful values for N_{ref} are:

0 : The three NGSs are used as references with an accuracy limited by the probe arm positioning error.

1 or 2 : Absolute reference is established, but no distortion modes can be corrected (based on the reference sources themselves, it might still be doable using the science objects). Tip/tilt correction in one direction is, in principle, possible for $N_{ref} = 2$. It is, however, irrelevant for differential astrometry.

Many : Absolute reference is established and plate-scale modes (and potentially higher-order distortions) can be corrected.

Not all combinations of $N_{science}$ and N_{ref} are meaningful. For example, $N_{science} = 1$ is only meaningful in combination with very small N_{ref} (0 or 1). If more reference sources were available, we would declare them “science objects” in the field (in the context of the terminology used here) which can be used for distortion correction. Also, distortions will generally be corrected using the science objects, if more than two are available, as their precision is likely much higher than that of the reference sources (this might be different for some science fields once Gaia data become available). Thus cases with large $N_{science}$ will, for the most part, give the same results for the distortion solutions for any value of N_{ref}, the latter only determining the accuracy of the absolute coordinate system.

The combinations considered in the error budget table of Fig. 1 are therefore $N_{science} = 1$ with $N_{ref} = 0$ and 1; and large $N_{science}$, in which case N_{ref} can be dealt with in parametric form.

2.3 Absolute astrometry accuracy

Absolute astrometry is the determination of the object positions in the sky coordinate system. We consider the same combinations of $N_{science}$ and N_{ref} as in the previous section, for the same reasons.

2.4 Error budget color coding

Many of the fields in the error budget spreadsheet are equations with adjustable input parameters, providing access to a large sample set of astrometry science cases. This is indicated in the figure through the background color coding:

White: Hard-coded value or text
Light purple: Field value is calculated from an equation. In the spreadsheet, the parameters used are also highlighted in purple, on the right in the same line. They are not shown here for space reasons.
Light blue: Field value is total of the preceding subcategories, added in quadrature.
Orange: Adjustable input values. They can be used to explore the effects of varying observation parameters. In the example shown, they are set for K-band observations of a field with many reference stars and high-signal to noise ratio, with the astrometric measurements of interest happening in a small portion of the field. An example of such a science case would be measuring orbits of the brighter stars at the Galactic Center (for which confusion is not a dominant term).

2.5 Special science case spreadsheets

The eight columns shown in Fig. 1 are an attempt to capture what we call generic science cases in the TMT astrometry error budget. Not all science cases of interest can be described in this general form. The spreadsheet therefore also contains additional sheets describing individual science cases of particular interest (not shown here).

3 Error Budget Categories

We have split up the TMT astrometry error budget into five categories, following roughly the light path from the science objects to the instrument detector plane.

3.1 Reference catalog errors

This category captures errors caused by the differences between the real and assumed properties of the references sources. Obviously, it does not affect differential astrometry precision, as no reference sources are used for that. This error category is dominated by errors in the positions and proper motions of the reference sources and strongly dependent on their number.

Errors or uncertainties in our knowledge of the reference source colors and their variability couple with atmospheric dispersion to produce astrometric errors. This is currently under investigation, but preliminary results show that it will not be a dominant error term in most cases. It might, however, lead to requirements for the magnitude of acceptable star color uncertainties for the highest-precision science cases.

The other or unknown motion category is meant to capture effects such as binary star motion or gravitational lensing. It is, by definition, a minor astrometry error source for the generic science cases, but might have to be considered for specific cases.

Aberration is the compression of the field (in one direction) due to the relative directions of the Earth's motion and the velocity vector of the light coming from the reference sources. This effect is epoch dependent in that the velocity vector of the Earth changes throughout the year. We have shown that this effect is deterministic and negligible once the lowest order terms have been removed in post processing.

A special situation occurs if extra-galactic non-point-source references (for example, globular clusters in/around distant galaxies) need to be used as reference sources. As this is a specialized case, it is not part of our standard astrometry error budget, but we might want to include an extra science case in which this is used. It is therefore listed as a separate line that is not added to the other terms. For the time being, we adapt the value from Ref. [5].

With the exception of color errors in combination with dispersion, our assessment of the errors in this category should be quite reliable, although we will revisit the derivation of some of them again when we finalize the error budget.

3.2 Atmospheric refraction correction errors

Atmospheric refraction enters the astrometry error budget because it alters the apparent position of the objects in the field. It has an achromatic differential term that is due to the differences in zenith angles of different objects in the field and a chromatic differential term due to the wavelength dependence of the index of refraction of air.

The achromatic term is mostly linear and therefore taken care of by coordinate transformation in post processing as long as enough reference sources are available. By contrast, the chromatic term is linked to the uncertainty of our knowledge of reference object colors (as described in the previous section) as well as of the profiles of atmospheric temperature, pressure and humidity. It requires both an atmospheric dispersion corrector (or correctors, tuned independently for different wavelength bands) and a posteriori corrections for the highest-precision astrometry.

Refraction-caused errors can be among the dominant terms in the TMT astrometry error budget. They are still under investigation in both simulations and measurements at Subaru Observatory.

3.3 Other atmospheric effects

This section covers error terms caused by atmospheric effects other than refraction. The first three terms are due to residual distortion caused by atmospheric turbulence after correction by the AO system. The fourth term is also caused by residual atmospheric turbulence, but quantifies the irregularity, and in particular the asymmetry, of the field-dependent PSF after AO correction. These four terms have been analyzed through detailed simulations of TMT's first-light AO system, NFIRAOS, and are well understood. They are small in magnitude due to the fact that NFIRAOS is an MCAO system and have been shown to depend on the integration time as $T^{-0.5}$ as is expected from theory.

In dense science fields, science objects may lie on the seeing-limited halos of other objects. Errors in the subtraction of the halos then cause background gradients that lead to astrometric errors. This is called the halo effect. Its magnitude is very strongly dependent on the star density of the field and the quality of the AO correction and can take on a large range of values.

Atmospheric effects that vary during an exposure, such as the Strehl ratio or atmospheric transparency, couple with any effect that causes image motion, for example time-varying distortions. Unintuitively, this error term increases with integration time due to the shape of the power spectra of atmospheric variability. This does, however, mean that the magnitude of the error can be controlled by keeping the individual science exposures short. The term in the error budget is therefore an allocation for the maximum allowable error caused by this effect.

Scintillation is a minor effect that is listed for completeness without a significant contribution to the error budget.

3.4 Opto-mechanical errors

This section lists astrometric error due to distortions in all parts of the opto-mechanics along the light path. The first term, guide probe positioning error, refers to the accuracy with which the guide probes for the natural guide stars of the MCAO system can be positioned. As there are three such guide probes, this error leads to stretching of the field and thus plate-scale errors. This can be a significant error source if no external reference sources are available. In fields with 3 or more references sources, it can be subtracted to a very high degree of accuracy, limited only by the signal-to-noise ratio of the set of reference source positions.

Distortions due to the imager optics, surface irregularities, the telescope optics, rotator errors etc. are much larger in absolute terms than the acceptable errors for TMT astrometry. However, as they are dominated by low-order modes, they can be reduced significantly through a combination of calibration measurements and post-processing (e.g. coordinate transformations). The stability of the distortions and the repeatability and reproducibility of the observational setup are therefore crucial for achieving the highest accuracy astrometric results.

Several of the terms in this section are currently the most uncertain sources of astrometric errors for TMT and are under active investigation. [7] So far, no show stoppers for differential astrometry on the level of tens of micro-arcsec have been found, but several tweaks to the NFIRAOS and IRIS designs have been made or suggested based on our analysis of these effects.

The vibration error budget for TMT is not fully understood yet. It is expected that vibrations matter more for other science cases than for astrometry and that they will therefore not be a major contributor to the TMT astrometry error budget. This has to be confirmed.

3.5 Focal plane errors

The first term in this section is the fundamental random astrometric error due to photon noise. It is proportional to the inverse square root of the integration time and can thus be reduced by integrating for a longer time. Note that this is the combined integration time of all individual exposures that are used for a given epoch. The value of 24 micro-arcsec in the table is for a S/N ratio of 200, which is approximately what will be achieved for a $K = 20$ point source with TMT/NFIRAOS/IRIS in a 100-second exposure.

Other errors in this section are due to different noise sources and detector properties. They are generally well understood through a combination of simulations and analytical analyses. The PSF estimation error still needs to be quantified, but our current analysis shows that it should not be a dominant error term in most cases due to the uniformity of the MCAO correction over the IRIS field.

Confusion errors are due to the influence of unresolved sources in the field on the estimated position of the PSF. By definition, this error is small for the generic science cases described here, because those are defined as cases with sufficient sources to do high-quality low-order distortion correction, but not so many that the halo effect or confusion become an issue. For very dense star fields, such as the immediate neighborhood of the Galactic Center, confusion might become one of the dominant error terms. [6] Mosaicing is marked as ’not applicable’ in this table for the same reason.

3.6 Total error

The total error in the last line of the error budget table shows that the TMT requirements for differential astrometry should, in principle, be achievable for some science cases. It is clear, however, from our analysis that this will only be possible with very careful calibration, observing and data analysis procedures. We also point out again that the number is highly dependent on the properties of the science field and can vary by large factor with respect to the ones shown here, both for the individual terms as well as for the total error.

4 Summary

This paper describes the current status of the TMT astrometry error budget work. It is based on results of analytical analyses, simulations and observations done by a large number of people and groups spread across the TMT partnership. We present a first, mostly complete version of the error budget table, although some of the values in it still need to be taken with care, especially since the magnitudes of the errors depend very strongly on the details of the astrometric observations. The astrometric precision or accuracy for a specific science case might therefore be very different from the values shown in Fig. 1. Nevertheless, we have so far found no show stopper for very high precision and accuracy astrometry with TMT.

Work on the error budget is continuing with the emphasis being on the highest risk and highest uncertainty terms. In parallel, we are also developing a mathematical formalism of observing sequences and data reduction processes. All of this will be combined into a comprehensive astrometry error budget report, the first version of which is expected to be released soon.

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